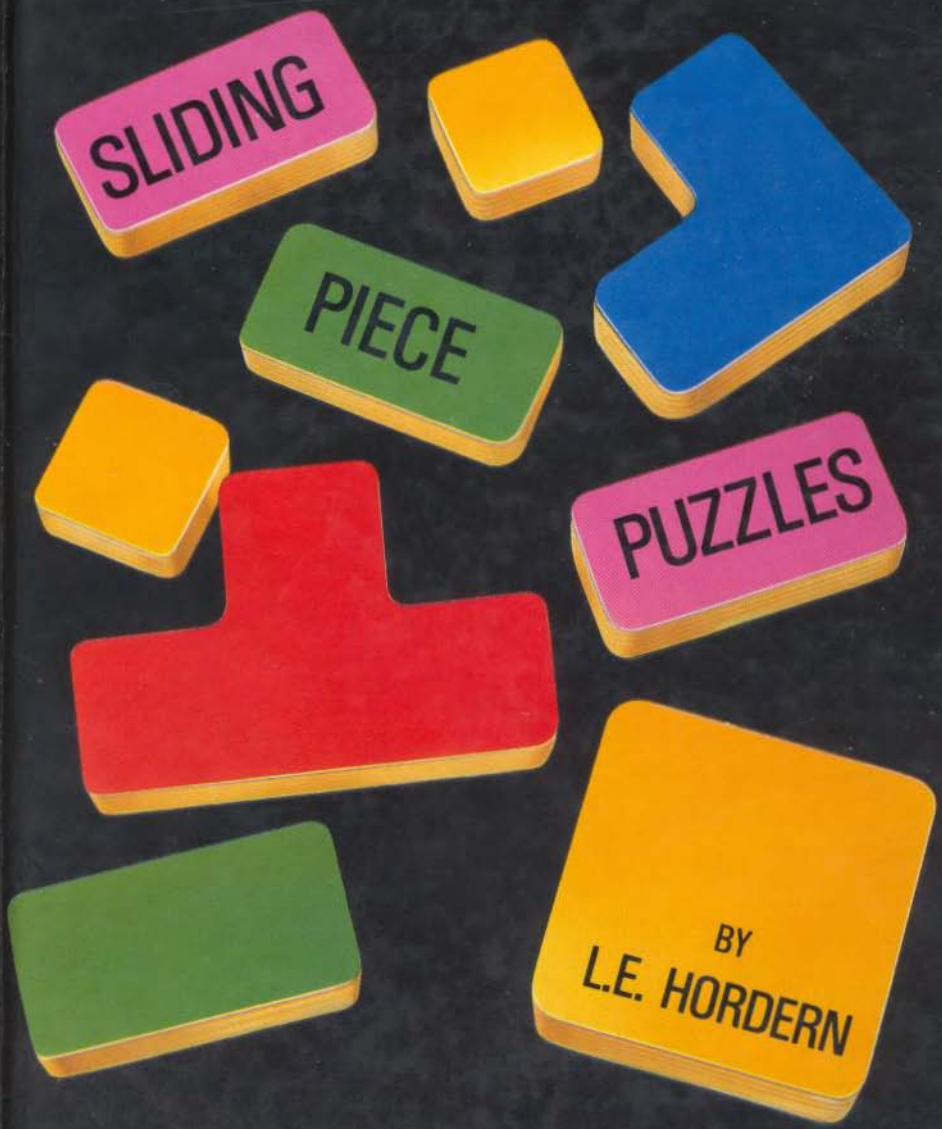
RECREATIONS IN MATHEMATICS



Recreations in Mathematics

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   The ins and outs of Peg Solitaire
3. Ernö Rubik, Tamás Varga, Gerzson Kéri, György Marx, and Tamás Vekerdy
   Rubik's cubic compendium
4. Edward Hordern
   Sliding piece puzzles

Sliding Piece Puzzles

Edward Hordern

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To Christopher, Mandy, Fern, and Sam

It's all very well for an hour or two
  But people might think it a bore
When after six weeks the solution one seeks
  Is just as far off as before.
The thing I've abused, Oh! the language I've used
  Half out of my mind I have been
And I'm rueing the day that they put in my way
  That horrible puzzle 'Fifteen'.

Reece (1880)

Foreword

David Singmaster

The sliding piece puzzle is one of the most popular of all mathematical recreations. A close relative—the famous Rubik's Cube—in 1980 spawned one of the two greatest puzzle crazes of all time. What is not so well known is that the other great puzzle craze—exactly a hundred years earlier involved a sliding piece puzzle called the 'Fifteen Puzzle'. In the ensuing century, there has been a remarkable number of different types that have appeared.

I first became aware of the profusion of such puzzles when I met Edward Hordern about five years ago and he gave me a copy of his booklet *150 Sliding Block Puzzles*. When this Series was first proposed, I immediately suggested that Hordern should update his booklet and this book is the result. He has significantly increased the number of sliding block puzzles described and has somewhat broadened the scope of the work to include shunting and switching puzzles and many other types of sliding piece puzzle, so that this book describes over 250 puzzles.

Edward Hordern, like many puzzle enthusiasts, has come to puzzles by a roundabout route. After some years in advertising and accounting, he started a small manufacturing firm which has prospered and gives him some time and the resources to pursue his hobbies. About fifteen years ago, he began to collect mechanical puzzles—that is, puzzles where a physical object must be manipulated in some way (though jigsaws are generally excluded). His collection is now one of the finest ever assembled, comprising some 8000 items, including many from the 19th century. Among these are about 750 sliding piece puzzles. He has an especial affinity for such puzzles and cheerfully spends hours, sometimes even years, finding better solutions for them. With his wealth of experience, I doubt if anyone can surpass his speed in solving a new puzzle. Most of the solutions given here have been found by him and many of these are notable improvements on previous solutions. In this book, Hordern shares his fascination and passion with the rest of us.

Preface

This is not a book for readers!

It is a collection of more than 270 sliding piece puzzles with the pieces provided for solving them. In the back of the book there is an envelope containing 'push out' pieces which can be used to solve some 230 of the puzzles. If the pieces should get lost or mislaid there is a 'back up' page of designs in Chapter 1 giving all shapes and quantities required of each piece.

There is a peculiar fascination in pushing blocks around a board—trying to get one to a particular position or attempting to form a certain pattern. Once started, people find them hard to put down. Often the only reason they don't pick them up in the first place is the fear of failure and the (quite unreasonable) thought that they might be made to look a fool or intellectually inferior. But the ability to solve sliding piece puzzles, like crosswords, has little to do with intelligence: some people find them easier to do than others. Just as there is a 'knack' to solving crosswords, so there is with sliding piece puzzles. Being able to think clearly and look ahead will help greatly. While the easier puzzles can often be solved by trial and error, the harder ones can rarely be solved this way. Considerable intuition and perseverance will be needed to crack the tough ones.

Whatever one's aptitude, the main purpose of this book is to provide entertainment. There is something for everyone: some puzzles are unbelievably easy, others so hard that few will solve them. And there are all those in between that are designed to give hours—perhaps even months—of fun.

Oxfordshire E.H.
April 1986

Acknowledgements

Many people from all over the world have helped in the preparation of this book. They fall broadly into three categories:

(a) Those who have given me help, information and advice.
(b) Those who have let me have details of puzzles in their collections.
(c) Those who have helped with some of the solutions.

Many people have let me have information and help on many puzzles and other information in this book and I thank them all. These include David Singmaster (England), Martin Gardner (USA), Nob Yoshigahara (Japan), James Dalgety (England), Dick Hess (USA), Trevor Truran (England), Stanislav Tvrdik (Czechoslovakia), Stewart Coffin (USA), David Pritchard (England), George Jelliss (England), and Dan Feldman (Israel). Many thanks to these and others who I have not mentioned.

I am most grateful to Jerry Slocum (USA), the late Eileen Scott (England), Abel Garcia (USA), and Will Strijbos (Netherlands) for having allowed me to include details of many antique (and modern) sliding block puzzles in their collections.

Last but by no means least I would thank those who have either improved on my solutions or who gave the best solutions in the first place: members of the Academy of Recreational Mathematics of Japan, Nob Yoshigahara (Japan), Kiyoshi Takizawa (Japan), Charles Levitt (South Africa), H. Moriya (Japan), T. Saito (Tokyo University), Len Gordon (USA), Ian Pedder (England), and K. Okada (Japan).

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Times Mirror Magazines Inc.—'Strategy' (1940) and 'Change the Seasons' (1942) from *Popular Science*.

United Media and World Almanac Publications—'Jeepers' from *Real puzzle book* by D. Rubin.

A. P. Watt Ltd.—'Set Up' from *Figure it out* by D. St. P. Barnard.

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1

ABOUT THE PUZZLES

The '15' puzzle

It all started just over 100 years ago; the world's first puzzle craze. The second one—almost exactly a hundred years later—was the Rubik's Cube phenomenon. There cannot be many educated people in the world today who have not heard of a Rubik's cube, nor handled one.

And so it was in 1879 and 1880, that a small square box carrying 15 little square blocks of wood captured the imagination of the world and caused at least as much of a stir as the Rubik's Cube. It was called the '15' Puzzle.

The 15 numbered blocks were to be put into the (4 × 4) square box at random. All the solver had to do was to slide them one at a time, without lifting or turning, so as to put them in order: 1–4 in the top row, and so on down to 13–15 in the last row, followed by the vacant space (See Figure 1).

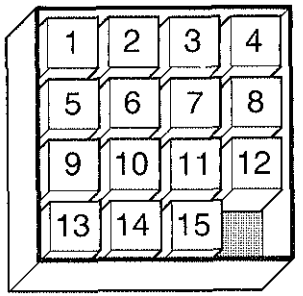


Figure 1

The great fascination of the puzzle was that sometimes it was solved very easily and at other times it would not yield to any amount of sliding of the blocks. It was possibly this element of uncertainty in something so apparently simple that made the puzzle so irresistible. It does not appear reasonable to the simple human mind that sometimes it

should work out and not others. One's natural instinct is to say that either it should always succeed or not at all.

From this explosive beginning has developed the multitude of sliding piece puzzles of all shapes and sizes, from the absurdly easy to the incredibly difficult.

Scope

There are four kinds of sliding piece puzzle, the first three of which are covered in this book:
(i) Sliding block puzzles—in which each piece can move independently of the others;
(ii) Soko (warehouse) puzzles—in which one piece must push all the others;
(iii) Railway shunting puzzles—in which one or two pieces must push or pull all the others;
(iv) Sliding puzzles with plungers or levers—in which pieces move in groups.

Category (iv) has been omitted because readers are unlikely to be able to construct them, and although they achieved a popularity a few years ago, they are largely unobtainable now. A selection is shown in Plates X and XI.

What all sliding piece puzzles have in common is (a) pieces that slide, without lifting or jumping, from one position to another, and (b) a space into which to slide the pieces.

The famous 15 Puzzle belongs to the sliding block category. There have been considerably more of this type than of all the others put together. They are certainly the more interesting to solve and their relative ease of manufacture has ensured their continued popularity.

Over the last 100 years so many sliding block puzzles have been produced and then forgotten about, and so few seem to be available today, that the idea came of assembling everything of interest and putting it into one book. However, as research progressed, it became evident that there was far too much material, and some other criterion was needed. It was decided not to include any puzzle unless it has at some time been manufactured or published in some form—published material includes books, magazines, scientific papers and patents. Certain puzzles have been included because of their historical value rather than their interest as puzzles; others, interesting as they may be, have had to be excluded, usually because of their similarity to puzzles already included. All the puzzles listed have been made in one form or another (most are in the author's collection), except for those where only a book, magazine or patent is shown as the source. It is not known whether these puzzles were ever produced. The sources of the books and magazines is given in Appendix C and a list of patents in Appendix B. The country of origin of all other puzzles is given alongside their titles. Where no country is shown, the puzzle emanated from the UK.

The chapters have been arranged according to the type of puzzle. Some people may find one type of puzzle more interesting than another. Consequently, when attempting the sliding block puzzles in Chapters 3 to 7, the reader is recommended to start with the puzzles in Chapter 5, which may have a more general appeal, before attempting those in Chapters 3 and 4.

It is often impossible to tell just by looking at it whether a puzzle is more worthwhile than others, and therefore a list of 'preferred' puzzles is given at the beginning of each chapter.

Difficulty ratings

As is so often the case with puzzles, some that look very easy to solve are in reality very difficult and vice versa. A grading system, rating the degree of difficulty of every puzzle, has been introduced to help the reader:

* easy; warm-up exercise;
** standard;
*** tricky;
**** difficult;
***** exceptionally difficult;
******the ultimate challenge.

Beginners are urged to start with puzzles rated * and ** and work their way upwards. Otherwise disillusion can set in quite quickly! Even readers with some experience of this type of puzzle are urged not to start with the **** and ***** puzzles, but to progress to them gradually. Be warned! Experts can take days or weeks to solve the most difficult ones. The author took four years to discover the insight leading to one solution—a puzzle he had previously thought impossible.

The object in some of the puzzles (mostly in Chapters 3 and 4—a few in Chapter 7) is to solve them in a specified number of moves. Just to solve the puzzle in unlimited moves is too easy. Therefore a second rating in brackets: '. . . in minimum moves' gives the rating

for solving the puzzles in the required number of moves. It is this rating that the reader should follow.

The ratings are necessarily subjective and one person's view, but it is hoped they will provide a reasonable guide.

Taxonomy

In the taxonomy of mechanical puzzles there are more than a dozen principal classes of puzzle. Examples include: 'disentanglement' (wire/wire and string puzzles), 'dexterity' (rolling ball puzzles) and 'route finding' (mazes). One of these categories is called 'sequential movement puzzles'.

The sequential movement category comprises three sub-classes:

(i) Rotational puzzles (includes the famous 'Rubik's Cube');
(ii) Sliding piece puzzles;
(iii) Peg jumping puzzles (includes the well-known European 'Solitaire' puzzle).

This book is about the middle category. Sliding piece puzzles can be further divided into:

(a) Sliding block puzzles (Chapters 3–9);
(b) Warehouse (Soko) puzzles (Chapter 10);
(c) Railway shunting puzzles (Chapter 11).
(d) Sliding puzzles with plungers or levers (Plates X and XI);

Identification

What is the definition of a sliding piece puzzle? A sliding piece puzzle consists of a number of pieces of any shape(s) enclosed within a defined area (or confined space), in which the purpose is either to rearrange the pieces into a predetermined order or to get a particular piece into a specified position. This is accomplished by sliding the pieces into areas not occupied by other pieces. In sliding block puzzles, all the movable pieces are capable of independent movement, but in most cases are not allowed to rotate; in warehouse (Soko) puzzles there is the restriction that none of the pieces can move except one, which must push (never pull) all the other pieces one at a time into their correct positions; in railway shunting puzzles there is the restriction that none of the pieces can move except one or two, which must push or pull the other pieces, singly or in groups, along specified routes into their correct order or places; and in the plunger/lever type puzzles the pieces can only be moved in groups by means of plungers or levers.

The difference between jumping peg puzzles and sliding piece puzzles is obvious, as the latter allows no jumping. However, the difference between the plunger/lever puzzles and rotational puzzles can appear to be rather obscure. The essential difference is in the existence of a space. A Rubik's Cube has no space and there should be no difficulty in seeing that it does not belong in the sliding piece category. However, the plunger/lever puzzles could be mistaken for rotational puzzles since they apparently have no space. In point of fact, the plunger type puzzles *do* have spaces even though they may be hidden from view or occupied (temporarily) by plungers, levers and/or springs. Although these spaces are outside what might be termed the 'working area' of the puzzle, they exist just the same. Pieces are moved into these spaces while other operations are performed. Therefore, in the strict sense of the word, they come into the definition of sliding piece puzzles.

Moves

Definitions of moves for Soko (warehouse) puzzles and railway shunting puzzles are given at the beginning of Chapters 10 and 11 respectively.

What constitutes a move in a sliding block puzzle? There are several possible definitions and, unfortunately, authorites cannot agree on a standard definition.

There are four possible alternatives:

1. Slide *one* piece only in any direction or combination of directions. The piece may be slid any permissible distance without lifting and (unless specifically allowed) without rotating. (Under this definition a piece can move around a corner; see Figure 2.)
2. Slide *one* piece only in any *one* direction. The piece may be slid any permissible distance without lifting or rotating (see Figure 3).
3. Slide any number of pieces together as a group in any *one* direction. Pieces may be slid any permissible distance without lifting or rotating (see Figure 4).
4. Slide *one* piece in any (orthogonal) direction, one unit (= the same distance as the dimension of the smallest piece; see Figure 5).

Definition (i) is the preferred definition for several reasons. The majority of puzzles produced have specified or intimated this definition where solutions have been given. It is the accepted

definitions of most (but not all) puzzle experts and collectors. The normal method of moving pieces will be with one's forefinger. It is logical that a move should start with the placing of the finger on the piece and end when it is taken off. It is irrelevant whether it has negotiated a corner in the meantime. If definition (ii) is used, an ambiguity occurs in situations where it is possible to move diagonally.

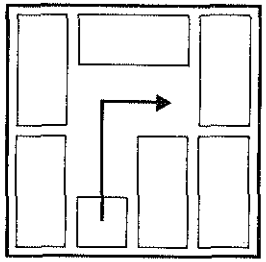


Figure 2

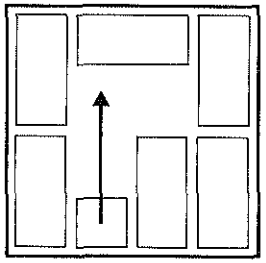


Figure 3

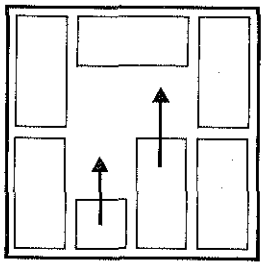


Figure 4

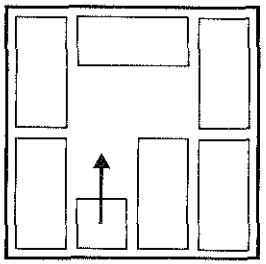


Figure 5

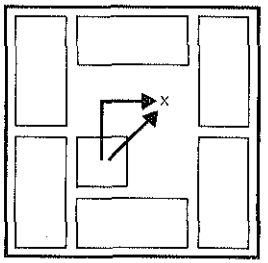


Figure 6

In Figure 6 the small square has the possibility of moving in two different ways to the spot marked X. If one allows only orthogonal movement, it would count as two separate moves under definition (ii); a diagonal move, on the other hand, would obviously only count as one. Under definition (i) no such confusion arises; it counts as one move however the piece is moved.

It might seem incredible that anyone could seriously propose definition (iii), which is the most awkward and unwieldy of them all. But in the 1930s a magazine specializing in games used it as their definition in an article about sliding block puzzles. More recently a manufacturer of a sliding block puzzle instructed solvers to use it too.

At the end of the description of each puzzle, the number of moves needed to solve it is shown. From Chapter 5 onwards it is given in the form: 'Solution: 76 moves (87)'. The first figure refers to the number

of move required under definition (i) and the second, which is always in brackets, refers to definition (ii). In the solutions section, where different from each other, solutions for both definitions are given. The only exception to this is for some of the difficult puzzles at the end of Chapter 6, where only solutions for definition (i) are given. The reason that definition (ii) solutions are omitted is that confusion may arise as to whether a diagonal move should count as one move or two.

Solutions

The solutions given at the end of the book are the shortest currently known. A handful of these have been proved to be minimal by computer. The majority of the remainder are thought to have the minimum moves. Some of the really hard puzzles are probably not minimal, but just to solve them is satisfaction enough.

The majority of the solutions have been done by hand by the author, and many have been improved (shortened). For example the original solution to Ma's Puzzle (D1) was given as 61 moves, but it is easily solvable in 23 moves. Similarly, the original solution to Get my Goat (C2) had 46 moves—this has now been reduced to 28. These unnecessarily long solutions are not confined to the older puzzles. Modern puzzles sold in the last few years with incredibly long solutions include Lost Pygmies (C70), whose given solution has 274 moves but can be done in 130, and Little Hippo (C63), where the given solution has 231 moves but needs only 84 moves. These are just a few of many. The original solutions to the above puzzles are so much longer than necessary that it is hard to believe that the solvers were really trying!

The complete answer to getting the shortest solutions is to use a computer. After much deliberation it was decided not to include a section on programming computers to solve sliding block puzzles, because it would take all the fun out of solving the puzzles. It would be like playing a game of chess and getting a computer to play for you—or doing a crossword and looking up the answers before one has looked at the clues.

Hints and tips

Several puzzles have been included that are not so much puzzles as 'warm-up' exercises for the more difficult ones. It is essential to

master the principles involved, because they recur frequently as part of other puzzles. For all the puzzles in Chapters 3 and 4, it is important to know how to exchange the positions of two adjacent pieces within a confined space. Puzzle B1 is the required 'exercise' (puzzles B19 and B20 are a good extention to this principle). Although more difficult, B46 is another good exercise in the manipulation of uniform pieces. When rectangular pieces are introduced, the first exercise is to solve C1. This will illustrate the principle that what at first sight appears to be impossible can be solved with a little perseverance.

Puzzles that have pieces with assorted sizes and shapes present quite different problems, and the most helpful pieces are always the small squares. These usually have to precede or follow the large square in order to facilitate its progress—especially when it comes to turning a corner. Good puzzles to illustrate the point are C19 and C52.

L-shaped pieces add a whole new (difficult) dimension and the best exercise is D4, which illustrates several principles.

For the more difficult puzzles, especially those in Chapter 6, it is quite useless sliding the pieces at random in the hope of getting somewhere. It is most important to develop a strategy. This should consist of a series of intermediary objectives along the following (much simplified) lines: piece *x* cannot be moved to *m* until piece *y* is moved out of the way; piece *y* is blocked by piece *z*, which must be moved first. . . Continuing on from this reasoning: piece *x* cannot get past piece *y* until pieces *r*, *s*, *t* are out of the way and pieces *a* and *b* are in their place. An example of tactical strategy would be to plan to get all the pieces into a certain order from which the puzzle can be solved. In many cases, it is the largest or most akwardly shaped piece (the key piece) that has to be moved to a specific location. This may involve rearranging almost all the other pieces of the puzzle before the key piece has moved at all. A further hint is to move L-shaped pieces together to form a rectangle and to note how they move easily round corners in some directions but not others.

If the more difficult puzzles are tackled on the basis that they are 'puzzles within puzzles' then they often readily yield to solution.

Parity

When the '15' Puzzle (see B10 in Chapter 4) was first produced, would-be solvers were intrigued to find that sometimes the puzzle

came out very easily and at other times it appeared to be unsolvable. The first mathematical papers showed that exactly half of all the possible random start positions led to a solution (and half did not). Attention was then focused on the mathematical laws that governed problems of this sort: was there a general rule that governed all puzzles of this kind, and was there an easy method of identifying whether a particular start position was solvable? The answer to both questions was, of course, yes.

The law governing problems of this type is commonly referred to as 'parity', and performing a 'parity check' is the means of identifying solvable and unsolvable positions. 'Changing the parity' of a puzzle means changing it from an unsolvable to a solvable state (or vice versa). The parity check method given below applies to a rectangular board of any shape or size providing it has uniform pieces and one space the same size as a piece.

1. Exchange any two pieces so that one or both pieces go to their correct position. Exchanging pieces means physically lifting them from their place—not normally allowed in solving the puzzles.
2. Repeat the exchanges until all the pieces end up in their correct positions.
3. Count the total number of exchanges; if the number is even, the puzzle is solvable from its original position; if the number is odd, it is not solvable. (The maximum number of exchanges is always one less than the number of pieces).

There is a mathematical theorem which shows that it does not matter how or in what order the exchanges are carried out—they may even be repeated—this test will always give the correct answer.

An easy way of counting the exchanges in, for example, the '15' puzzle, is to count a 'cycle' of exchanges, as follows:

(a) Pick up piece No. 1 and put it in its correct place, removing the number already there.

(b) Repeat (a) with the new number.

(c) Continue until a number goes into a vacant space, which ends the cycle. Count the number of moves in the cycle. The number of exchanges is always one less than the number in the cycle.

(d) If there remain numbers not positioned correctly, start a new cycle and continue until all numbers are correctly placed. (With a little practice this method can be done in one's head).

The parity principle is used to effect (to confuse the solver) in certain puzzles in Chapters 3 and 4 and in one or two more puzzles in other chapters. It would appear from a parity check that these puzzles

can only be solved if parity can be changed. But is this possible? If so, how?

To go any further would be to reveal an important part of some of the solutions to these puzzles. Therefore, so as not to spoil the enjoyment for those who wish to tackle them without referring to the solutions, this discussion on parity is continued at the beginning of the solutions section.

Making the puzzles

Two alternatives for making the puzzles are offered, depending on whether the reader wants 'instant' puzzles, or, with only a little time and effort, a set of superior pieces.

Alternative A

A set of 53 pieces on a pre-cut card is given in an envelope at the back of the book. The same pieces are reproduced in Figure 7 in case some get lost. The pieces in the envelope may be readily separated from each other and can be numbered or lettered, if required, so that the solutions can be followed. Where coloured pieces are needed, they can be lettered: B (blue), R (red) etc. For the railway shunting puzzles in Chapter 11, it is best to use the 2 × 1 rectangular pieces (where 11 or fewer are required). The direction of the engines/coaches can be shown by marking them with an arrow. It is strongly recommended that a pencil is used to mark the pieces, as marks can then be erased easily and the pieces re-marked for a different puzzle.

The only further item that will be necessary is a board, tray, enclosure or box, in which to slide the pieces. The simplest board is made by drawing a thick black line with a felt tip pen on paper, card or any other suitable smooth surface to mark the outline of a particular puzzle. Obstacles and immovable blocks can also be marked in this way. It helps greatly in solving the puzzles if the pieces and the board are of different colours. For this reason, the pieces in the back of this book have a colour tint.

When drawing a thick black line to mark the border for a selected puzzle, bear in mind that it needs to be slightly larger than the area of the pieces. In puzzles having a square or rectangular border an extra margin of one quarter of the smallest dimension of the smallest piece is recommended. For example, if the smallest piece is a square having a 12 mm side, then the extra margin around the edges should be

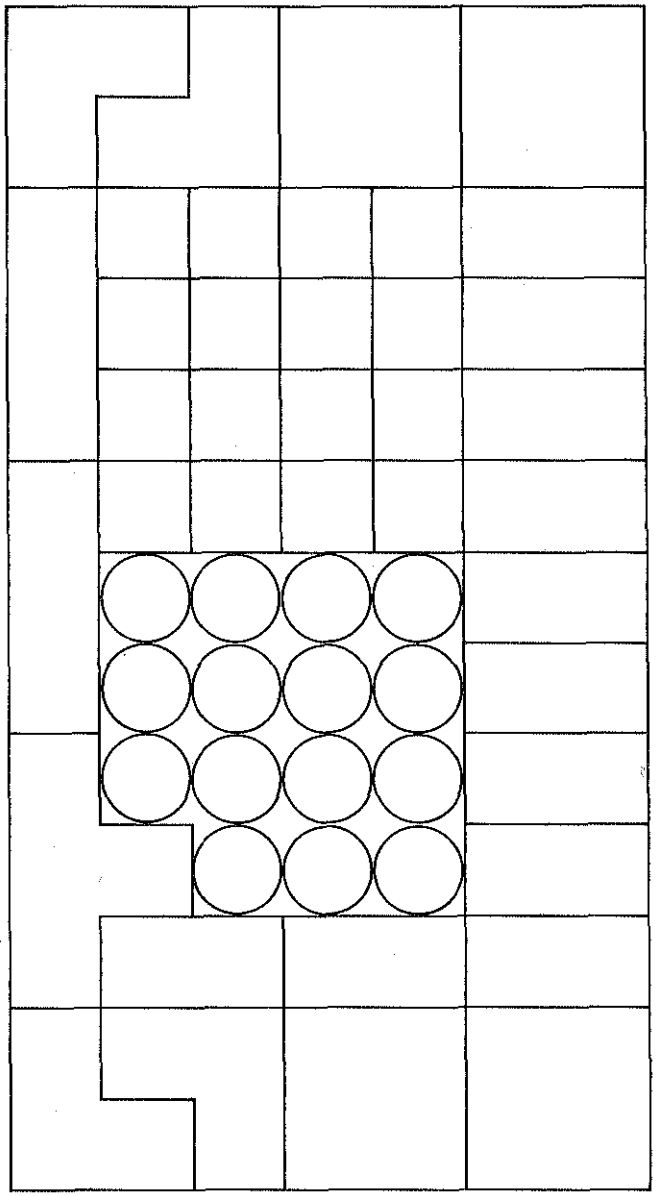


Figure 7

about 3 mm. A felt-tip pen can also be used to make a board consisting of circles connected by a series of lines.

By drawing the outlines of the puzzles on a suitable surface and using the pieces in the envelope provided, the reader will be able to tackle some 230 of the puzzles or more—see Table 2.

The pieces at the back serve a most useful purpose in providing 'instant puzzles'. But they suffer from two disadvantages. For reasons of space, they have had to be made far smaller than the ideal, and consequently they are rather fiddly to manipulate. Another drawback is that they have square corners, which have an annoying tendency to catch on neighbouring pieces when being slid from one place to another. Both these disadvantages can be overcome by making one's own pieces.

Alternative B

Since most people shudder at the thought of making anything, especially if it sounds the least bit complicated, every effort has been made to allow readers to make their own pieces with the minimum effort. The pieces listed in Table 1 are reproduced full-size as Figures 8 and 9 in the colour plate section: Figure 8 is the basic set and Figure 9 the supplementary set (see Table 2). Just take four photocopies of

Table 1: Piece shapes and sizes

	Shape	*Unit size*[a]
a	T shape	3 × 2 . . . less two non-adjacent (1 × 1) corners
b	Long L	4 × 2 . . . less one (3 × 1) corner
c	Large L	2 × 2 . . . less one (1 × 1) corner
d	Medium L	2 × 1 . . . less one (1 × ½) corner
e	Small L	1 × 1 . . . less one (½ × ½) corner
f	Large Square	2 × 2
g	Small Square	1 × 1
h	Rectangle	4 × 1
i	Rectangle	3 × 1
j	Rectangle	2 × 1
k	Rectangle	1½ × 1
l	Circle	1 . . . diameter
m	½ hexagon	any

[a] It is recommended that each unit should be approximately 20 mm.

each set, stick each complete photocopy onto thick card or board and then cut out the shapes with scissors or a sharp knife. Half an hour or so with glue and scissors is all that is required. If no photocopier is available, tracings can be used instead.

The simplest board can be made in the same way as in Alternative A, but it is more satisfactory to make proper trays, so that the pieces slide about within a defined area or 'enclosure'. Full design details for a set of wooden trays are given below, but they can just as well be made by gluing pieces of wood or card onto a suitably smooth surface.

The basic set of piece shapes in Figure 8, made up into four sets, provides all the same pieces that are in the back of the book, and a few extra that are not needed. For example, only one T-piece is needed, whereas the photocopying produces four. Table 2 gives all the pieces that are required, so that time need not be wasted making unnecessary pieces. The same 230 puzzles (or more) can be attempted with the basic set as with the ones provided at the back.

A supplementary set of piece shapes is reproduced in Figure 9, and these consist of unusual shapes that are used only for a few puzzles, as well as extra pieces of the basic set needed for some of the larger puzzles. Four photocopies of this set are also required. When used in conjunction with the basic set, they will allow the reader to tackle a further 17 puzzles—a total of 247.

It should not be beyond the scope of the enthusiast to make their own pieces for F6–F8 and F10–F12. The remaining puzzles not covered in Table 2 cannot be attempted as their boards are far too complex or they are three-dimensional.

Those readers who wish to solve the maximum amount of puzzles yet make the minimum number of pieces are recommended to make the following 38 pieces: one T shape, four large L pieces, four large squares (2 × 2), sixteen small squares (1 × 1), two rectangular (3 × 1) pieces, and eleven rectangular (2 × 1) pieces—most of the basic set. The 230 puzzles shown in Table 2 can still be tackled if the squares are used instead of the circles.

The best pieces of all will be made out of wood, plastic or other suitable material. The designs in Figure 8 and 9 can be photocopied and used as templates for cutting out the pieces. Whatever pieces are made, ensure that they have rounded corners—it really *is* worth the extra trouble and makes the difference between an ordinary puzzle and one that is a joy to manipulate. The radius of the curve should be one eighth to one tenth of the smallest dimension of the smallest

Table 2: Piece sets and the puzzles they will do

Piece	Complete set needed for 247 puzzles	Pieces provided in envelope	Basic set [Figure 8 (× 4)]	Supplementary set [Figure 9 (× 4)]	Basic + supplementary (× 4)
a T shape	1	1	1 → 4[a]		4
b Long L	2			1 → 4[b]	4
c Large L	6	4	1 → 4	1 → 4[b]	8
d Medium L	4			2 → 8[d]	8
e Small L	4			1 → 4	4
f 2 × 2	4	4	1 → 4		4
g 1 × 1	35	16	4 → 16	5 → 20[c]	36
h 4 × 1	4			1 → 4	4
i 3 × 1	2	2	1 → 4[b]		4
j 2 × 1	11	11	3 → 12[c]		12
k 1½ × 1	7			2 → 8[c]	8
l Circle	15	15	4 → 16[c]		16
m ½ hexagon	16			4 → 16	16
	111	53	15 → 60	17 → 68	128
Surplus pieces	–	–	7	10	17
TOTAL PIECES	111	53	53	58	111
No. of puzzles possible	247	230	230	N/A	247

[a] Includes three surplus pieces. [b] Includes two surplus pieces. [c] Includes one surplus piece. [d] Includes four surplus pieces—note that there are left-handed and right-handed versions of this piece.

With either the pieces provided in the envelope at the back of the book or the Basic Set [Figure 8 (× 4)] all the puzzles in the book are possible *except* A3, B22, B26, B40, C5, D5–D8, D49, E10, E11, E15–17, E33, E35, F1–F13, G1–G5, H4, H8, H10, J5, J6, J12, J13. If the circular pieces are used as additional squares, a further six puzzles can be attempted: B22, B26, B40, H4, H8, J5.

Puzzles possible with the basic and supplementary sets together [Figures 8 and 9 (× 4)]: all the puzzles in the book *except* E10, E11, E15–E17, E33, E35, F3–F13, G1–G5, J12, J13.

piece. The ideal dimensions of the pieces themselves are such that the smallest piece (square) should be slightly larger than the tip of one's forefinger. As in Figures 8 and 9 this would mean a unit size of about 20 mm—making the small square 20 × 20 mm. This could be reduced to about 15 × 15 mm, but any smaller and the pieces become difficult to slide easily.

It is very helpful in solving the puzzles if the pieces have different colours: all the small squares one colour, all the large squares a second colour, all the vertical rectangles a third colour, all the horizontal rectangles a fourth colour, etc. This makes it much easier to visualize what pieces are where and where they should go. The author has often used Lego pieces, which come in a variety of shapes (including L shapes), colours and sizes, for just such a reason. They are unsuitable in many respects (sharp corners, too small) but are instantly available and their colours make solving difficult puzzles that much easier.

Drawing lines on a card to delineate the boundaries of the playing area is all very well but it is far from ideal. There is a world of difference between pushing pieces around a board that consists of a black line on a piece of paper, and a well-made sliding block puzzle with a proper raised border, where the pieces slide smoothly and do not catch on one another. If the reader is making his own pieces from wood or some other material, then it should not be too difficult to construct a tray or two with edges. It is suggested that two trays and six 'filler' blocks are constructed. With these it will be possible to adapt one or other of the trays to any of the 'board sizes' for the puzzles in Table 2. The tray sizes should theoretically be 10 × 7 units and 6 × 6 units (1 unit = the small square), but they will need to be slightly larger to accomodate the movement of the pieces. If the corners of the pieces are nicely rounded then the extra margin at the border of the trays should be about one eighth of the smallest dimension of the small square—about 2½–3 mm if the small square is 20 × 20 mm. If the corners are not rounded, then this margin should be doubled. The theoretical dimensions of the 'filler' blocks in units are: 10 × 1, 6 × 1, 5 × 1, 4 × 1, 3 × 1, 2 × 1. The long dimension of each should be slightly more than the theoretical so as to take up the extra space created by the margin. This is especially important with the 10 × 1 and 6 × 1 blocks, which will need to be 'jammed' between the two sides of the puzzle (not necessarily along an edge). Pieces can also be used as filler blocks where necessary. By jamming two of the filler blocks, one crosswise between the sides and one lengthwise between the top and the first filler block, it is possible to construct almost any size of board.

The base of the tray, on which the pieces are to slide, should be as smooth as possible. If made of wood, the surface should be sanded and either waxed, polished or lacquered.

It is a good idea, but not essential, to make lids for the trays so as to prevent pieces falling out or getting lost.

Thus with only two trays, six filler blocks and a choice of 53 or 111 pieces one has all the equipment needed to undertake some 230–247 puzzles.

The best-made puzzles that the author has seen are those made by Minoru Abe (see Plate VII bottom left—also puzzles D24–D49). Pieces from these puzzles are shown on the front cover of this book. The base is a metallic looking plastic; the rest of the tray and the pieces are made of chunky plywood; the pieces are coloured according to shape and have rounded corners; and the puzzles are large enough to be a joy to manoeuvre. The only possible (very small) criticism is that there is no lid or box to put the puzzle in. In terms of handling, this series is far superior to anything else ever produced. Mr Abe lives in the northernmost part of the Japanese mainland where he runs a place called 'Coffee Shop Now'. If he gives his puzzles, which are extremely difficult, to his customers while they are drinking coffee, he must sell coffee by the litre! It conjures up the picture of people sitting there weeks later still pushing little blocks of wood around, and having their 1000th cup of coffee!

2

HISTORY OF THE SLIDING BLOCK PUZZLE

History of the '15' and '14–15' puzzles

(A full list of references is given in Appendix B)

Who invented the sliding block puzzle (in particular the '15' Puzzle) and when? Unfortunately, authorities cannot agree on either.

During the latter part of the last century and the first part of this century, two people dominated the world of puzzles: Sam Loyd (1841–1911) in America and Henry Dudeney (1847–1930) in England. They each invented thousands of puzzles (of all kinds) and they also 'borrowed' heavily from each other. The same puzzles, in different guises, keep cropping up in their books. There is no doubt that Sam Loyd invented many puzzles, some of which became 'mini-crazes' in their own right. But none had the world attention that was given the '15' Puzzle.

Dudeney (1926) maintains: 'But the great crazes only became possible under quite modern conditions. The first notable case was that of Loyd's 'fifteen' puzzle, that in 1873 was sold by the million and for a short time almost monopolized the attention of Europe and America . . . and the world positively "went mad" over this little thing'. Loyd himself (1914) says: 'The older inhabitants of Puzzleland will remember how in the early seventies I drove the entire world crazy over a little box of movable blocks which became known as the 14–15 Puzzle'.

But there is a wealth of evidence to show that not only was that date wrong but that Sam Loyd never invented it. If the above wording is looked at more closely, he makes no claim on its invention—only that 'he drove the world crazy'. Many chroniclers, especially the later ones, assumed that Loyd invented it, presumably because of his long association with puzzles and inventing them.

Plate I. Clockwise from top left: Rate Your Mind, Pal (A1); Four Square (A10); La Grande Question (A5); Superpuzzle 100—an example of an extremely difficult puzzle because all the pieces are so nearly the same; Ten Little Nigger Boys (A6); The Premier (A2).

Plate II. Left to right: Au revoir/Do svidania (A8); Archie's Puzzle of the Heads (A9); Craps! (A9); Le Moulin Rouge (A7).

Plate III. Top row (left to right): Time Puzzle (B50); Parka Car (B35). Bottom row: Vanish Mystery Puzzle (B29); Peyo Switchit (B34).

Plate V. Top row (left to right): Relax (C27); Mickey and Minnie Compact Puzzle (C22); Lost Pygmies (C69). Bottom Row: (Broken Heart) Reverse Puzzle (C27); Mintman Puzzle Mints (C27); Qwik-Sane (C3).

Plate IV. Clockwise from top left: Boss, Game of the Fifteen (B10); Black and White (B44); The New '15' Puzzle (B60); The So-Easy (B41); The Teaser (B42).

Plate VI. Clockwise from top left: Get My Goat (C2); Dad's Puzzler (C19); The Infants Hospital Puzzle (C12); George Washington Puzzle (C67); Kapture The Kron Prinz (C2).

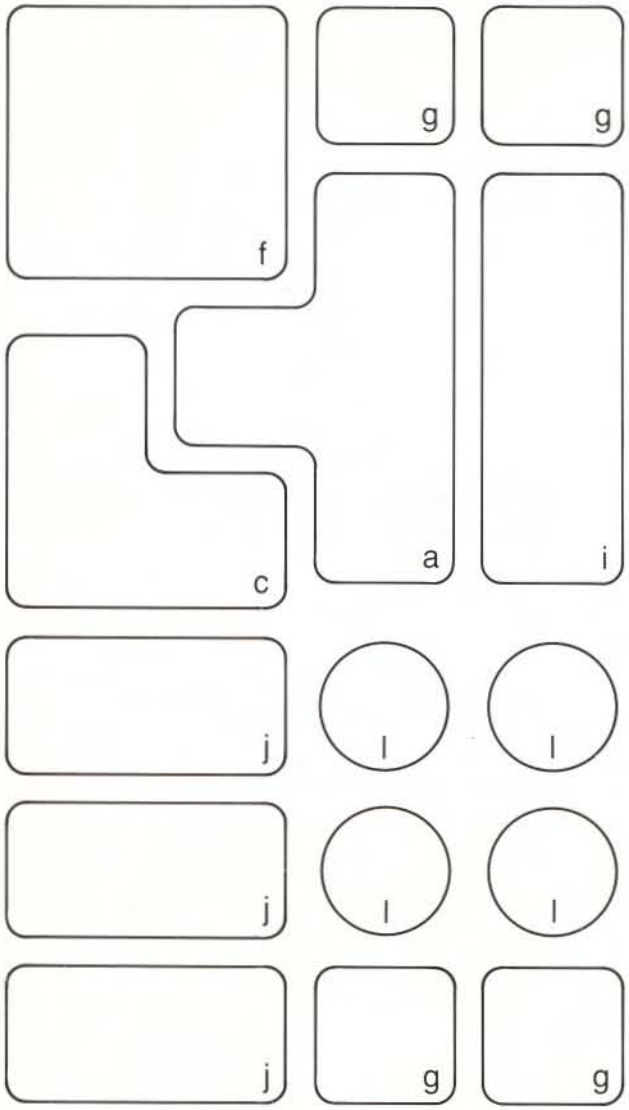


Figure 8

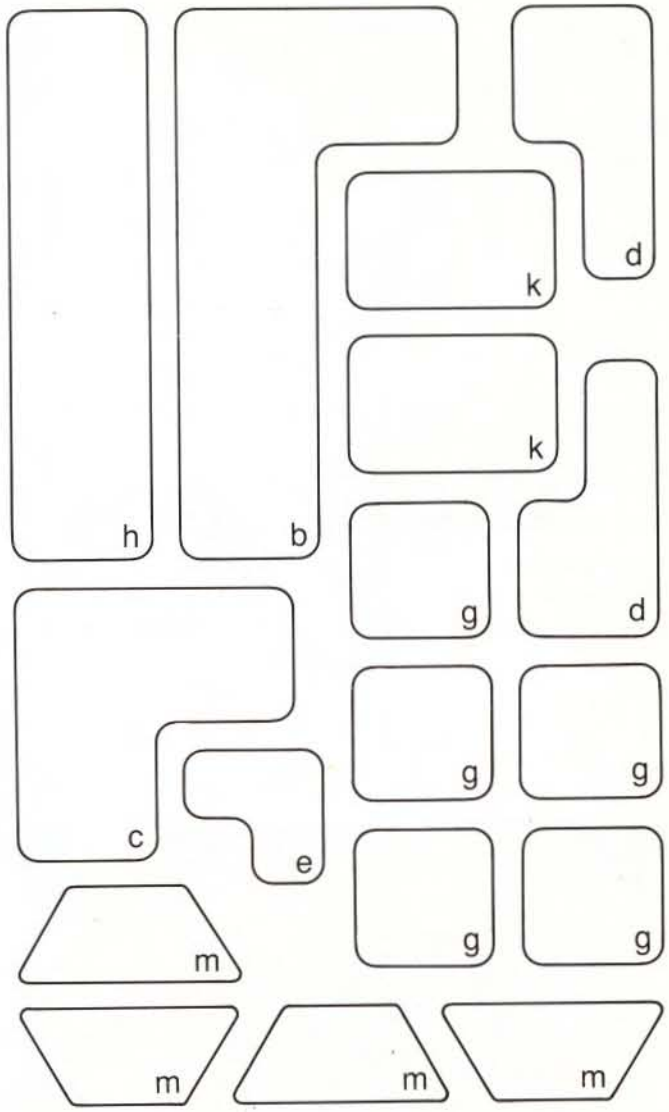


Figure 9

Plate VII. Clockwise from top left: The Traffic Jam Puzzle (D12); Comic Scramble Game (E20); The Klondyke Miners' Puzzle (E14); Ma's Puzzle (D1); Climb Game 15 (D45).

Plate VIII. Top row (left to right): Orbit (E6); Manoeuvre (E30); Inter-City (E29). Bottom row: Chifu-Chemulpo (E7); Automobile (E11); Perplexity (E10); Puzzle—Get Protection into . . . (F5).

Plate IX. Three-dimensional sliding block puzzles. Front row (left to bottom): Qrazy Qube (G2); Varikon Box 'L (G3); Mad Marbles (G3). Back row (top to right): Change the Seasons (G1); Pionir Cube (G4); Inversion (G3).

Plate X. Top row (left to right)—various 3-D sliding puzzles: Puzzle Pen; 6 by 6 (numbered); 6 by 6; Missing Link; Popeye Can Puzzle; Babylon Tower; Entrapment; 6 by 6 (with beads). Bottom row—various sliding puzzles with levers: 8-Pole Wisdom Plate; Row by Row; Rack 'em up; Great Gears.

Plate XI. Various sliding puzzles with plungers or levers. Top row (left to right): Hiroko; Four by Four; 5 × 5; Ten billion. Bottom row: Uriblock; Trillion; Crossover.

Plate XII. Railway shunting puzzles. Top row (left to right): Turntable Train (J13); Good Luck Railroad (J7); Whistle Stop Puzzle (J4). Bottom row: Aiguillages (J11); Shunting! (J13); Pacific 231 (J3).

Loyd and Dudeney made their name in puzzles partly because they were supreme at inventing and solving them; but an essential factor in their success was also their ability to 'dress up' an otherwise uninteresting mathematical exercise by embellishing it with stories and anecdotes. Many of these were dreamed up for the purpose—especially where it came to the origin of certain puzzles. In his *Eighth book of Tan* (1903, republished 1968), Loyd perpetrated an enormous hoax on his readers about the origin of the Tangram. Unfortunately, even the eminent philologist, Sir James Murray was taken in, and went to considerable lengths only to find there was no basis in Loyd's story. It is best described by Peter van Note in his introduction to the Dover edition (Loyd 1968) of the same book.

> He chose the title, he explained, because some four thousand years ago a Chinaman (or Chinese God) named Tan had compiled seven books of Tangram patterns. . . 'The seven books of Tan', said Loyd, 'were supposed to illustrate the creation of the world and the origin of the species upon a plan which out-Darwins Darwin, the progress of the human race being traced through seven stages of development up to a mysterious state which is too lunatic for serious consideration'. With that statement scholars like British philologist Sir James Murray should have been warned. In his own special way Loyd blended half-truths and popular suppositions about things Oriental with a few figments of his own imagination; to this he added a few well-dropped names, and with a twinkle in his eye served the old puzzle up in a delightful mock-serious stew. Had Sir James looked more closely, he might have realized that Loyd's whole essay was 'too lunatic for serious consideration'. Instead Sir James embarked on a search for the historical Tan. . . (Loyd 1968).

While the thought of China being scoured by one of Sir James' relatives on a wild goose chase is very amusing, it is far from helpful to the historian, and casts doubt on other statements of fact made by Loyd.

It is important to realize that there were two puzzles—not just one: the '14–15' Puzzle was quite different from the '15' Puzzle. The '15' puzzle has a 50 per cent chance of success—the '14–15' puzzle is impossible. Loyd (1914) further says: 'The fifteen blocks were arranged in the square box in regular order, only with the 14 and 15 reversed. The puzzle consisted in moving the blocks about, one at a time, so as to bring them back to the present position in every respect except that the error in the 14 and 15 must be corrected.'

This is quite different from the random placing of the blocks required for the '15' Puzzle, as described by all the early chroniclers.

Johnson (1879): 'A ruled square of 16 compartments is numbered. . . Fifteen counters, numbered in like manner are placed at random upon squares so that one square is vacant. . . The puzzle is to bring all the counters into their proper squares by successive moves'. Sala (1880): 'You take out the number 16; you mix up the counters in the box so that they will run irregularly'. Snowdon (1880): 'Sixteen small wooden cubes, numbered from one to sixteen, are placed in random order in a shallow box'. The instruction booklet entitled 'Albert Durer's Game of the Thirty-Four and Boss—Game of the Fifteen' produced by Cremer of 210 Regent Street, London in the spring of 1880 gives a long description and proof (?) of how 'only half the possible combinations can succeed'. This is clearly a reference to the random placing of pieces—not Loyd's 14–15. Kirkman (1880), Tait (1880), Warren (1880), Lucas (1881, 1882–94), and Schubert (1909) all refer to the random placing of pieces (i.e. the '15' Puzzle, not the '14–15 Puzzle). None of the above mention Sam Loyd.

Lucas, Schubert, and Ahrens are more specific and give dates as well. Lucas (1881) says that Mr Sylvester, the English mathematician, correspondent of the 'Académie des Science de Paris' and professor at the John Hopkins University of Baltimore, USA, told him the '15' puzzle had been invented by a deaf mute 18 months previously. The same article, with subtle amendments, was printed in the second edition of his book (Lucas 1882–1894), and one of the changes was the date. This now became 'towards the end of 1878'. Schubert (1909) gives the date as December 1878. Ahrens (1918) says that Sam Loyd is supposed to have invented the '15' puzzle but also quotes the deaf mute story mentioned above and says that it is a legend. He says the only certainty is that the puzzle was invented in America.

Loyd's son, Sam Loyd Jr., does not agree with his father about the date: 'It was in the early eighties when I had barely attained my 'teens that the world-disturbing "14–15 Puzzle" flashed across the horizon and Loyds were amongst its earliest victims. To say that I was infatuated with the tantalizing box of blocks is a mild description of my enthralment' (Loyd 1928). Would he have written in this manner if his father had invented it?

The dates and the extent of the world's first big puzzle craze are fairly well documented. It started in America in 1879 and rapidly spread to Europe. The editors of the *American Journal of Mathematics* stated: 'The "Fifteen" puzzle for the last few weeks has been prominently before the American public, and may safely be said to have engaged the attention of nine out of ten persons of both sexes

and of all ages and conditions of the community' (Johnson 1879; Story 1879). Snowdon (1880) goes on to say: 'What then has caused this puzzle to become so immensely popular? How does it happen that one dealer in New York is said to have sold 230 gross (33 120) of a cheap variety of this puzzle in one day? Why is it that when once the puzzle is taken up no account is taken of the flight of time? That at the first attempt the figures may perhaps be soon shifted into the required consecutive order, while over the next random arrangement hours may be spent without the desired result being obtained?'

Proctor (1881b): 'It is singular to think that probably not fewer than twenty millions of persons tried The Boss Puzzle'.

Sala (1880):

> A short Act of Parliament should be passed prohibiting, under penalty of heavy fine and long imprisonment, all and sundry of her Majesty's subjects from playing at a dreadful game called 'Fifteen' which is known in the United States as 'The Great Boss Puzzle'. . . But pshaw! what need have I to describe the fearsome game? Even as I write, thousands of my readers, old and young, may be playing it. If time be indeed money, that Great Boss Puzzle must have cost me at least a thousand dollars between January and June last. I played it at Omaha; I played it at Chicago; I played it at Great Salt Lake City; I played it on board the Hecla coming home; and upon my word, so soon as I have finished writing "Echoes", I shall be at the Great Boss Puzzle again. Why was it not stopped at the Custom-house? Why was it not brought under the provisions of the Dangerous Explosives or the Cattle Plague Laws? There would be no use in proceeding against the persons who have naturalised this appalling apparatus in England. Our old friend 'the merest schoolboy' can make a game of Fifteen for himself from so many buttons or draught-counters. It is the players who, in the interests of Precious Time, should be punished.

This passage in the *Illustrated London News* of 22 May 1880 gives us an accurate dating. Since Sala refers to 'between January and June last', it can only be interpreted as referring to 1879. This confirms other writers' dates of the craze starting in 1879 in America.

Lucas (1881) says that several months after its appearance in America the 'Jeu du Taquin' was imported into France and offered for sale in political and illustrated journals under the name of 'le double casse-tête gaulois'. He goes on to say that its success in Europe was perhaps even greater than in America.

Schubert (1909) says that the puzzle appeared in 1879 and 1880 in Germany under the title of 'Boss' puzzle, in England as the 'Fifteenth Puzzle' and in France as the 'Jeu du Taquin'. Ahrens (1918) says that

it was introduced into Germany in 1878. He also tells of Deputies in the Reichstag (Parliament) in 1880 not paying attention to the proceedings as they were too occupied with the Boss Puzzle.

Dudeney (1926): 'Certain London shops in Cheapside and elsewhere sold nothing else and were besieged from morning to night, while hawkers at every street found it impossible to supply the demand'.

They even wrote songs about it: 'THE FIFTEEN PUZZLE—sung with immense success by J. J. DALLAS in the Burlesque Drama "THE FORTY THIEVES" at the Gaiety Theatre, (London) composed by GEORGE MEEN, written by MR. REECE' (*Punch*, 8 January 1881; Meen and Reece 1881). Part of Mr Reece's song is reproduced on the dedication page at the front of this book.

Loyd (1928): '. . . puzzle history's most notable event. . . The 14–15 puzzle craze did not come gradually. . . Instead, it burst upon our unsuspecting globe as might a meteor out of the sky. And the reverberations of its arrival spread with almost the speed of light to the uttermost corners of the world'.

> People became infatuated with the puzzle and ludicrous tales are told of shopkeepers who neglected to open their stores; of a distinguished clergyman who stood under a street lamp all through a wintry night trying to recall the way he had performed the feat. The mysterious feature of the puzzle is that no one seems to be able to recall the sequence of moves whereby they feel sure they succeeded in solving the puzzle. Pilots are said to have wrecked their ships; engineers rush their trains past stations and business generally became demoralised. A famous Baltimore editor tells how he went for his noon lunch and was discovered by his frantic staff long past midnight pushing little pieces of pie around on a plate. (Loyd 1914).

If Loyd did not invent the '15' puzzle, then just what was his contribution? There seems little doubt that he added in no small way to its popularity. First, he decided on a particular arrangement of the pieces (instead of asking the solver to place them at random in the tray); and then he set the task of exchanging pieces 14 and 15, while, of course, ending up with all the other pieces in their original positions. He might have chosen any two single pieces—the 1 and the 15, for example—because any such single exchange is impossible. Knowing that his puzzle was impossible he offered the enormous sum of $ 1000 to anyone who could provide a solution. 'A prize of $ 1000, which was offered for the first correct solution to the problem, has never been claimed, although there are thousands of persons who say

they performed the required feat' (Loyd 1914). In an interview in 1907 Loyd again says:

> '. . . in spite of that, however, there are thousands of persons in the United States who believe they solved that puzzle . . . but the thousand dollars reward I offered for anyone who could do it was never claimed. Not long ago the Sunday editor of a New York paper wanted to use it again as a supplement, and I suggested he should offer a thousand dollars reward for the solution. He refused. He said he remembered very well that he had done the puzzle once and he was not going to throw away a thousand dollars. Before I could persuade him to offer the reward, I had to bring the thousand dollars to his office and deposit it in the safe. It was never claimed.' (Bain 1907).

Loyd must have achieved considerable publicity. Anyone who offered the equivalent today (it must be at least $ 25 000) would be assured of fairly extensive media coverage. A good recent example was that of Kit Williams and his Golden Hare, which achieved both national press and television news status.

What is not so clear is just when Loyd offered his reward. We can only conjecture that it was some time in 1880 or 1881. Proctor (1881c) wrote on 25 November 1881:

> A prize is said to have been offered in America to anyone who should bring the blocks into this position—called the won position—starting from a position differing only from the 'won position' in having the three blocks in the fourth line arranged 13, 15, 14 instead of 13, 14, 15 (a position which has been called the 'lost position'), and thousands wasted hours on hours of their time in the attempt to do this impossible thing. Some said they had done it, but were assuredly mistaken. Others thought they had satisfied the conditions of the problem by getting some such arrangements as. . . [Refer to puzzles B11, B12. (Chapter 4).]

Loyd claims that he tried to get a patent on the '14–15' Puzzle:

> Mr Loyd has patented and copyrighted many of his inventions, but failed to get a patent on the 'Fourteen-Fifteen' puzzle. 'Of course, it couldn't be done', said Mr Loyd, 'and that's why I did not get my patent. It was necessary then to file with an application for a patent a "working model" of the device. When I applied for a patent, they asked me if it was possible to change the relations of the fourteen and fifteen. I said that it was mathematically impossible to do so'. 'Then', said the commissioner, 'you can't have a patent. For if the thing won't work, how can you file a working model of it?' 'His logic was alright, and the result was that I did not get my patent'. (Bain 1907).

By 1881, it seems that the craze was almost over. On 11 November 1881, Proctor wrote: 'I thought the Fifteen puzzle was dead, and hoped I had had some share in killing the time-absorbing monster. [It is an excellent puzzle by the way. . .]' (Proctor 1881b).

Much later Dudeney refers to the '14–15' Puzzle: 'And yet today it is dead as Queen Anne. Mathematicians set to work on the thing and discovered a rigid proof of its impossibility. When this became known it gave the thing a death blow, and the craze died as suddenly as it arose'. Dudeney (1926).

Whether this was true or not for the '14–15' Puzzle, the public does not seem to have taken much notice of the mathematicians when it came to the earlier '15' Puzzle. The first proof—that half of all possible positions were solvable and half were not—came in 1879 in America from the *American Journal of Mathematics* (Johnson 1879; Story 1879). In 1880 the first proof appeared in the UK in the *Royal Society of Edinburgh Proceedings* (Tait 1880). Both these proofs also contained details of how to tell whether a random pattern was solvable or not. But the method proposed was rather laborious. It was simplified by Lucas, who in 1881 produced the first proof in France in the *Revue Scientifique de la France et de l'Étranger* (Lucas 1881). The instruction leaflet to Albert Durer's Boss Puzzle by Cremer, already mentioned, has a proof of sorts. None of these proofs seems to have dented the popularity of the puzzle, which appears to have continued unabated until about the middle of 1881.

The simplest way of determining whether a random pattern of the pieces is solvable or not is simply to count the number of exchanges necessary to get the pieces into their correct positions. If the count is even, it is solvable; if it is odd, it is impossible. With a little practice this can be done in one's head. It seems that this was realized (and even published) quite early on and a novel way of *always* solving the puzzle is suggested by Warren in *The Nation*:

1. Tip out all the pieces onto a table and make sure they are all face up.
2. Close your eyes and mix up all the pieces.
3. Put all the pieces back into the box, still keeping your eyes closed. Naturally some pieces will face left, right or away from you.
4. Open your eyes and immediately count the exchanges necessary to solve the puzzle (the maximum is 14).
5. If the number is even rotate the individual pieces so that they all face you—and then solve the puzzle. However if the number is odd, first rotate the whole box 90° clockwise (or anticlockwise), then rotate the individual pieces and solve the puzzle as before.
6. As an added refinement the turning of the whole box can be avoided by turning instead piece 6 so that it becomes 9, and 9 so that it becomes 6. This will only be necessary when the count is odd and can be done in the course of righting all the other pieces. (Warren 1880).

Dozens of modern authors have all assumed that Loyd invented the '15' Puzzle. Few seem to have produced any evidence. Once two or three make the claim, it easily becomes self-perpetuating. It would be natural to think, from reading Loyd (1914) and Dudeney (1926), that Loyd had invented the puzzle in the early 1870s. Slocum (1986) is the most recent. He adds that he has a puzzle entitled: 'Embossed puzzle of Fifteen'—manufactured by the Embossing Co. which has printed on the lid, 'Patented Oct. 24, 1865'. The author has an identical puzzle, but Slocum confirms that a search at the U.S. Patent Office has revealed that no patent was issued to this company on that date.

Three things need to be remembered: (i) what was said by Loyd (1914) and Dudeney (1926) was published some thirty and forty years after the event; (ii) Sam Loyd's son does not agree with his father on the date (Loyd 1928) and writes in a way that makes it difficult to conclude that his father invented it; (iii) there is no contemporary evidence to corroborate either a date of the early 1870s or that Loyd invented it.

On the contrary, there is ample evidence in the years 1879–81 to suggest that the '15' Puzzle (not the 14–15 puzzle) created a world-wide sensation during 1879 and 1880. Contemporary evidence also suggests that the 14–15 puzzle appeared in 1881 (or possibly 1880). It is not certain whether Sam Loyd proposed the 14–15 version or someone else. But he did achieve notoriety by the offer of a large reward to anyone who could solve it.

From all the information available, it would seem that the likely course of events was that the '15' Puzzle was invented in America towards the end of 1878 by an unknown person (not Sam Loyd). The resulting puzzle craze started in early 1879 and spread to Europe in the same year. Sometime later, in either 1880 or early 1881, Sam Loyd proposed his version, the '14–15' puzzle, which gained him immediate notoriety through the enormous reward offered for its solution. The craze then seems to have died sometime in 1881.

The last word on the '15' Puzzle must go to the editors of the *American Journal of Mathematics*, who felt that they had to add some justification for the inclusion of the articles by Johnson and Story:

> But this would not have weighed with the editors to induce them to insert articles upon such a subject in the *American Journal of Mathematics*, but for the fact that the principle of the game has its root in what all mathematicians of the present day are aware constitutes the most subtle and characteristic conception of modern algebra, viz: the law of dichotomy applicable to the separation of the terms of every complete system of permutations into two natural and indefeasible groups, a law of the inner world of thought, which may be said to prefigure the polar relation of left and right-handed screws or of objects in space and their reflexions in a mirror. (Johnson 1879; Story 1879).

History of other sliding block puzzles

Much of the history of other types of sliding block puzzle can be traced through reference to patents both in the UK and the USA. A full list of patents is given in Appendix B. All patent numbers in this Chapter refer to US patents unless otherwise stated.

The '15' Puzzle achieved so much notoriety that it is tempting to believe that it was the first sliding block puzzle ever produced. However, there were two patents in respect of sliding block puzzles in the USA that appeared before the '15' Puzzle in 1878/1879. The first, in 1869, was by Edward E. Gilbert (No. 91 737), and had the name Alphabet Instruction Puzzle (see Figure 10). In the design, the pieces were kept in place by an overhanging lip, shown by a dotted line in Figure 10.

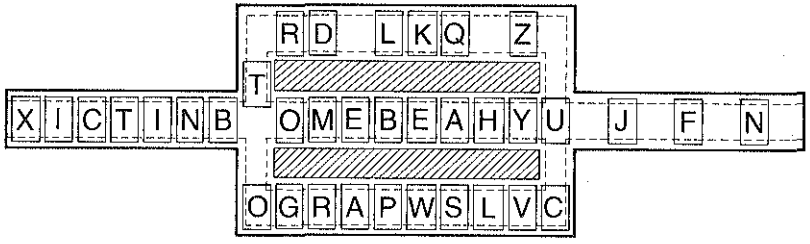


Figure 10

The idea of the puzzle was that it would help children to spell words; at the same time the pieces could not be lost. It is not known whether this design was ever produced, but many others on the same theme were sold during the last half of the nineteenth century. They

were mostly designed as educational aids, and have almost no value as puzzles.

Nine years later, in August 1878, Ernest Kinsey was granted a patent (No. 207 124) for a 6 × 6 square sliding block puzzle in which all the pieces were tongued and grooved to prevent any of them falling out. Although this patent also envisages the use of letters on the blocks, it was the first to suggest a single space into which another block could be slid. The inventor obviously had no faith in the solution of his puzzle because he provided for the frame to have a hinge or metal corner piece with pins or screws. This would enable the puzzle to be opened 'for more rapid arrangement of the blocks'! This patent also suggested triangular or diamond-shaped (parallelogram) pieces.

In May of 1880, Mary T. Foote was granted a patent (No. 227 159) in respect of 12 trays of multiplication tables for children. Each tray was a 3 × 12 sliding block puzzle in which the child has to arrange the table correctly e.g. 1 6 6; 2 6 12 . . . 12 6 72.

Also in 1880 but in England, Tait, in his proof of the '15' puzzle, is the first to suggest a three-dimensional sliding block puzzle: 'The principle above stated is, of course, easily applicable to the conceivable, but scarcely realisable case of a rectangular arrangement of equal cubes with one vacant space'. How wrong he was to be (see puzzle G2). Nine years later C. I. Rice in America patented just such a design (for a 2^3 cube)—No 416 344.

The original design for puzzle E34 (Chapter 7) was patented (No. 347 596) in 1886 by A. B. Harris. The first to convert the old problem of arranging different colours or card suits into different rows and columns was J. W. Brown (Patent No 433 444) in 1890—see puzzle A9, Chapter 3.

An interesting puzzle involving octagonal pieces in a 2 × 5 array, with a 'gate' that opened and closed, is suggested by C. A. McFadden Jr. (Patent No. 509 934) in 1893.

The first person to suggest rectangular pieces was H. Walton (Patent No. 516 035) in 1894—see puzzle C1, Chapter 5. But this puzzle had only rectangular pieces (2 × 1 shape). The idea of mixing square and rectangular pieces came in the same year in Patent No. 526 544 by L. D. Shriver. The invention relates to a 5 × 4 array in which there are 15 (1 × 1) squares and 2 (2 × 1) rectangles. As there is only one (1 × 1) space the most the rectangular pieces can do is to move lengthwise. It does not make a very interesting puzzle. The first 'proper' puzzle with a mixture of square and rectangular blocks,

allowing full movement of all pieces, was not patented until 1901 (No. 668 386) by F. E. Moss—see C14, Chapter 5.

Going back to 1896, we find an incredible design for a sliding block puzzle, which has an elaborate shape and a swinging lever that rotates on a pin. The patent was by W. E. Dow (No. 560 197)—see puzzle F13, Chapter 8. This puzzle had two additional 'firsts'. Of all the patents mentioned so far, it is not known whether any were actually manufactured at the time. This one *was* manufactured and, amazingly, made of plastic. It must have been one of the first items ever made in plastic. Needless to say, at that time (*ca.* 1895) the plastic making process had not been perfected and the plastic tended to be rather brittle and flaked if handled roughly. The puzzle is called the Combination Lock Puzzle and differs from the patent slightly in that it has five counters rather than four.

In the same year as the Dow patent R. J. Murphy introduced a design for a sliding block puzzle that included a turntable (No. 564 022). D. du Bois (Patent No. 590 093) produced another design including a turntable in the following year. Neither puzzle is very interesting. Another puzzle with a mechanical contrivance was issued in 1899 (Patent No. 619 804) to O. Svanström. This puzzle has a slidable tray inside it and can best be described as a forerunner (by 84 years!) to 'Row by Row' made by Tomy in 1983 (see Plate X).

The first puzzle to include 'L-shaped' pieces was designed in 1904 (Patent No. 771 514) by W. H. E. Wehner—see D5, Chapter 6. It is not known whether this puzzle was ever produced. The earliest-known puzzle of this type to be manufactured and sold was Ma's Puzzle in about 1927—see D1, Chapter 6.

Another puzzle that may never have been produced was the first design to allow rotation of the pieces. This extremely simple puzzle—see Nos. F7 and F8, Chapter 8—was patented in 1906 (Patent No. 811 321) by E. C. Pfeiffer.

L. W. Hardy seems to have been the inventor of the second most sold of all sliding block puzzles. According to Gardner (1975) he copyrighted it in 1909 and it was originally sold under the name Pennant Puzzle—see puzzle C19, Chapter 5. It is a puzzle that is most deserving of its long and continued popularity. The design is such that the objective can so nearly be reached, but then just when you think the puzzle is about to be solved, you become hopelessly stuck. The only way to solve it is to back track a little, get the two little squares all the way round the puzzle to the opposite end—and

then finally it can be solved. Although this puzzle was never patented, Hardy did patent another design. The patent was filed in 1907 (before the Pennant Puzzle) but the patent was not granted until 1912 (Patent No. 1 017 752). Why it took five years, we will probably never know. It is a very difficult puzzle—see puzzle C43, Chapter 5—which is probably why it may never have been sold.

Between 1927 and 1935 there seems to have been a revival of sliding block puzzles, a few of which were very difficult. Most of them conformed to a standard pattern of wooden pieces in a cardboard box. It was during this period that the third most sold sliding block puzzle was invented and produced. It is not certain whether it originally came from England or France, where it has always been known as L'âne rouge (The red donkey)—see puzzle C27, Chapter 5. It was patented in England in 1932 (Patent No. 411 515) by J. H. Fleming and has remained popular ever since.

In the same year and still in England, M. Bennett suggested a puzzle with triangular pieces (Patent No. 411 916) but it was probably never produced.

There was then an enormous gap—from 1935 until 1957—between sliding block patents in the USA. None were issued during this period and there have only been another 10 or so since. However during the 1970s and early 1980s there has been a revival—many old ones reproduced and a whole new range of two- and three-dimensional sliding block puzzles with mechanical contrivances. Since most of these emanated from Japan and Taiwan there is no evidence in the patent records.

One more patent is worth mentioning—No. 4 036 503 of 1977, M. L. Golick proposes a $3 \times 2 \times 1$ three-dimensional puzzle inside a transparent case. There are five blocks, but the novelty is that there is provision for rotating individual blocks.

Like all things, the popularity of sliding block puzzles has gone in cycles. The big boom of 1879 to 1881 was followed by a period of general interest in puzzles (sliding block and other types) lasting until about 1905. Then there was very little interest until the mid-1920s. A mini-resurgence occurred during the last half of the 1920s, which lasted until the mid-1930s. Another long break was followed by a further period of interest between 1975 and 1983.

It should not be forgotten that even during the periods of lesser activity, the old favourites have kept on appearing in the shops, often with different names and styles. It is interesting to note that the popularity of the '15' Puzzle has been maintained throughout most of

the last hundred years. Since World War II shops have nearly always had them or their equivalent in stock. Admittedly, they were mostly made of cheap plastic, but they have continued to be there just the same. Surely this makes the sliding block puzzle—out of all the various types of mechanical puzzles—one of the most universal and long lasting recreations in the history of toys and games.

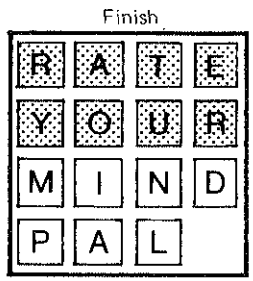
3

RANDOM ARRANGEMENT SLIDING BLOCK PUZZLES, A1–A10

This type of sliding block puzzle is by far the most common—and (usually) by far the least interesting. The two characteristics of the type of puzzle are that they have no set start position and that their pieces are all the same shape—square or circular. Thousands of this type of puzzle have been produced during the last forty years, the great majority made of plastic and with some picture or design on the pieces. Nearly all of these puzzles are extremely easy to solve and are probably intended for children, but every now and again one comes which is quite difficult. An example is the Superpuzzle 100 (10 × 10) shown at the bottom right of Plate I.

A1, A8, A9, and A10 are the most interesting puzzles in this section.

A1



Title: **Rate Your Mind Pal**—USA (see Plate I). Also in *Scientific American book of mathematical puzzles and diversions* (Gardner 1959)

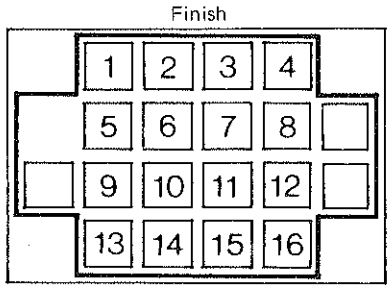
Date: 1940s or 1950s.

Start position: Random.

Object: Finish as shown. But there is a way to mix up the pieces that makes it difficult for another person to solve. How?

Rating: **

A2



Title: **The Premier**—the Prince of Puzzles—Gladstone and Beaconsfield.
Two 'sister' puzzles, one showing a portrait of Gladstone and the other Beaconsfield (see Plate I).

Date: Most probably 1880. It is possible to date this puzzle with some accuracy because Benjamin Disraeli became Lord Beaconsfield in 1876 and died in 1881. It is unlikely that it was produced before the great '15' puzzle craze reached this country (1879/1880) and equally it seems improbable that it was sold after Beaconsfield's crushing defeat in the 1880 election. It seems evident, therefore, that it was produced in the run up to the 1880 election. This must have been one of the earliest sliding block puzzles after the '15' puzzle and perhaps the first double sided one.

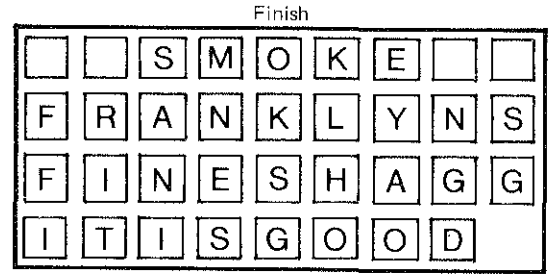
Pieces: One puzzle has 16 blocks showing a picture of Gladstone on one side, and letters on the other side which make the words: THE PREMIER PUZZLE. The other puzzle has a picture of Beaconsfield. In both puzzles the bottom right block must be removed to start the puzzle.

Start position: Random.

Object: To restore the picture.

Rating: *

A3



Title: **Franklyns Tobacco Puzzle.**

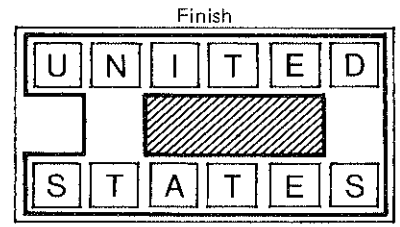
Date: 1920s or 1930s.

Start position: Random.

Object: Solution as shown in the diagram.

Rating: *

A4



Title: **United States Puzzle.**

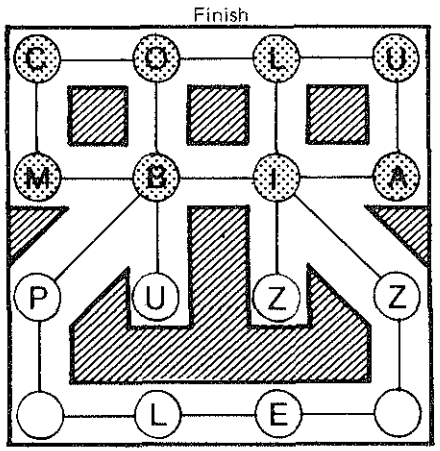
Date: Probably 1940s or 1950s. A US patent (No. 604 248) by F. D. Hopley differs only in that it has two extra pieces: one on the end of UNITED and the other on the end of STATES.

Start position: Random.

Object: Solution as shown in the diagram. Note immovable central block (shaded area).

Rating: *

A5



Titles: **Columbia Puzzle**—USA; **La Grande Question**—manufactured by N. K. Atlas, France (see Plate I).

Dates: 1895–1905 (both).

Pieces: Eight counters with the letters COLUMBIA on a dark background and six counters with the letters PUZZLE on a light background (Columbia Puzzle); eight counters with the letters QUESTION on a dark background and six counters with the letters DU JOUR on a light background (La Grande Question).

Start position: Place the eight dark counters at random on the eight bottom circles and the six light counters at random on any six of the top eight circles.

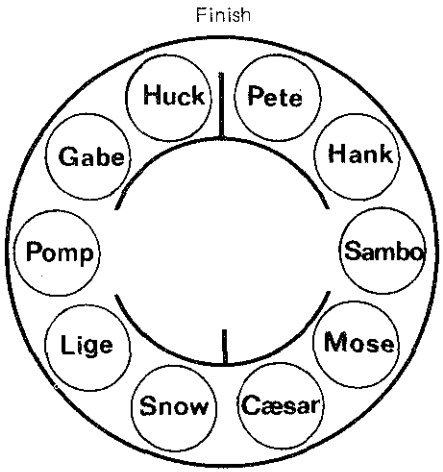
Object: Solution as shown in the diagram. Counters may only be moved from circle to circle along the lines. Note immovable blocks (shaded areas).

Rating: *

A6

Title: **Ten Little Nigger Boys** (see Plate I).

Date: Patented in 1898 by F. H. Donaldson, USA, (No. 602 735). Also RD No. 220 369. An earlier patent (No. 437 932) by T. W. McGrath in 1890 was not very different.

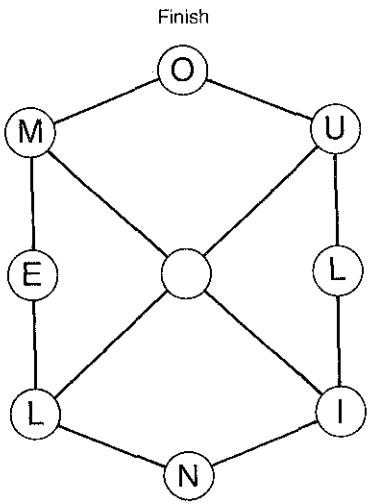


Start position: Random around the outside.

Object: Finish as shown in the diagram. Note: Two counters cannot pass each other in the inner circle.

Rating: *

A7



Titles: **Le Moulin Rouge**—manufactured by Simonin-Cuny, France (see Plate II); **Autour du Moulin**—manufactured by N. K. Atlas, France.

Dates: 1900–1904.

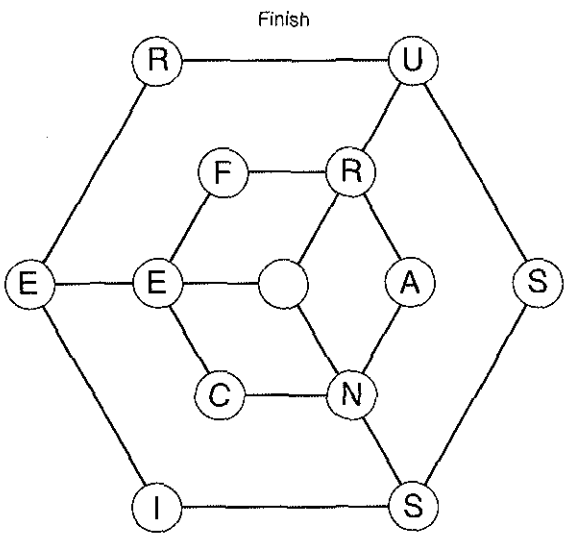
Pieces: Counters with the letters LE MOULIN.

Start position: Place the counters at random round the outer circle.

Object: Move all the counters to their correct position as shown in the diagram. Counters may only be moved *along the straight lines to a vacant circle.* Why is this always possible?

Rating: *

A8



Title: **Au revoir, Do svidánia**—manufactured by Watilliaux, Paris (see Plate II).

Date: 1893. It is evident that this puzzle was produced in celebration of the Franco-Russian Alliance of 1893.

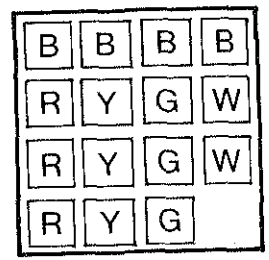
Pieces: Counters: six red with the letters FRANCE and six yellow with the letters RUSSIE.

Start position: Place the 12 counters anywhere at random but not on the centre circle.

Object: Move all the counters to their correct positions (yellow ones on the outside ring, red ones on the inside ring) as shown in the diagram. Counters may only be moved along the straight lines to a vacant circle. Why is it always possible to solve this puzzle?

Rating: *

A9



Titles: **Archie's Puzzle of the Heads; Craps**—USA; plus many others (see Plate II).

Dates: The principle was originally patented in 1890 (US Patent No. 433 444) by J. W. Brown. (Archie's and Craps—1940s or 1950s).

Pieces: Four blue, three red, three yellow, three green, two white. Alternatively any numbers or patterns can be substituted for the colours.

Start position: Random.

Object: Arrange the pieces so that no two of the same colour touch orthogonally or diagonally. The space should be at a corner.

Rating: ***

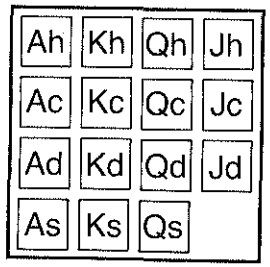
A10

Title: **Four Square**—from *Games Digest*, USA—a sliding block version of a much older puzzle (see Plate I).

Date: August 1938.

Pieces: Represent playing cards: A = ace, K = king, Q = queen, J = jack, h = hearts, c = clubs, d = diamonds, s = spades.

Start



Start position: Random.

Object: Arrange the pieces so that every row and column and the two main diagonals contain one of each suit and one of each value (ace, king etc.).

Rating: **

4

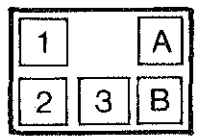
SLIDING BLOCK PUZZLES WITH UNIFORMLY SHAPED PIECES, B1–B60

The puzzles in this category look the same as those in the last chapter. The difference is that they all have a specific position from which to start. Solving these puzzles without regard to the number of moves is relatively easy, and most of the puzzles have a * (sometimes **) for this purpose. The real puzzle is to solve them in the stated number of moves and a rating: '(. . . in minimum moves)' is given after each normal rating. It is this that the reader should follow.

The most interesting puzzles in this group are: B7, B8, B9, B18, B24, B25, B27, B28, B31, B32, B35, B41, B44, B46, and if you like them difficult, B60.

B1

Start



Titles: **The Moving Day Puzzle**—from the *Cyclopedia of 5000 puzzles, tricks and conundrums* by S. Loyd; **A Lodging House Difficulty**—from *Amusements in mathematics* by H. E. Dudeney; **5 Block Puzzle**—from *100 puzzles: how to make and how to solve them* by A. Filipiak.

Dates: 1914 (Loyd); 1917 (Dudeney); 1942 (Filipiak).

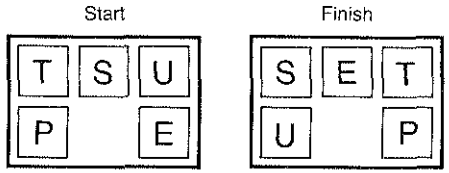
Start position: As shown in the diagram.

Object: Exchange the positions of pieces A and B.

Solution: 17 moves.

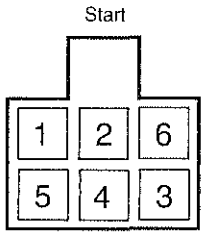
Rating: *

B2



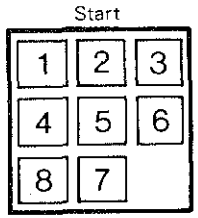
Title: **Set-up**—from *Figure it out* by D. St P. Barnard.
Date: 1973.
Start position: As shown in the diagram.
Object: Move the piece to the Finish position as illustrated.
Solution: 16 moves.
Comment: This is not so much a puzzle just to solve but to find the minimum move solution.
Rating: *

B3



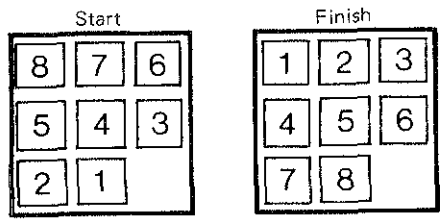
Title: **Bull Pen**—from *Games and puzzles you can make yourself* by Harvey Weiss.
Date: 1977.
Start position: As shown in the diagram.
Object: To get the pieces (Bulls) to their correct places (pens) which should be: left to right in the top row 1, 2, 3 and left to right in bottom row 4, 5, 6.
Solution: 22 moves.
Comment: A variation on the previous puzzle.
Rating: *

B4



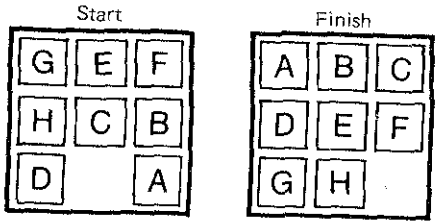
Title: None—submitted to *Amateur Handicraft Magazine*, USA, by F. Clark Hughes.
Date: 1947.
Start position: As shown in the diagram.
Object: Exchange the positions of pieces 7 and 8.
Solution: Impossible, if other pieces remain as shown. However if wording is altered and the object is 'to put all the numbers in their correct order' then it is possible. Two solutions are given.
Rating: *

B5



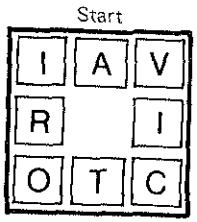
Title: **The Eight Block Puzzle**—from *Sixth book of mathematical games from Scientific American*, by M. Gardner. Based on an earlier puzzle by H. E. Dudeney.
Date: 1965.
Start position: As shown in the diagram.
Object: Finish as shown in the diagram.
Solution: 30 moves.
Rating: * (** in minimum moves).

B6



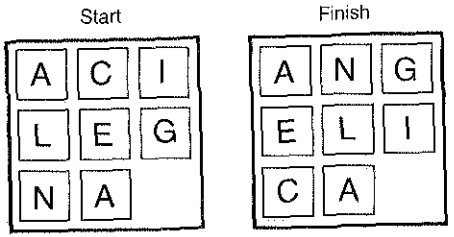
Titles: **The Letter Block Puzzle**—from *Amusements in mathematics* by H. E. Dudeney. Also: **The Baron's Puzzle**—one of a series under the general title 'Magnetic Norman'.
Dates: 1917 or earlier (Letter Block); 1984 (Baron's).
Start position: As shown in the diagram.
Object: Finish as shown in the diagram.
Solution: 23 moves.
Rating: * (** in minimum moves).

B7



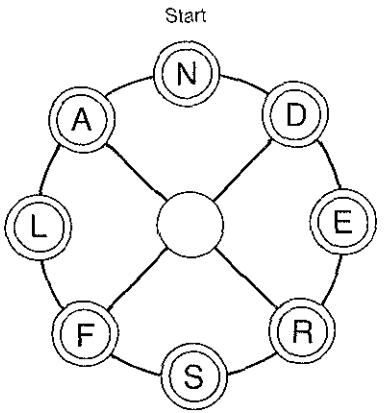
Title: **Victoria Cross Puzzle**—from *Amusements in mathematics* by H. E. Dudeney.
Date: 1917 or earlier.
Start position: As shown in the diagram.
Object: Make the letters read VICTORIA in a clockwise direction but with piece V not in a corner.
Solution: 18 moves.
Rating: * (** in minimum moves).

B8



Title: **The Angelica Puzzle**—from *Puzzles and curious problems* by H. E. Dudeney.
Date: 1917 or earlier.
Start position: As shown in the diagram.
Object: Finish as shown in the diagram.
Solution: 30 moves.
Rating: **

B9

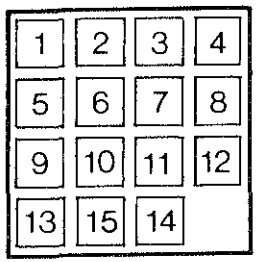


Titles: **The Flanders Wheel**—from *Puzzles and curious problems* by H. E. Dudeney. Also **The Asteroid Puzzle**—one of a series of under the general title of 'Magnetic Norman' (uses the word ASTEROID instead of FLANDERS).
Dates: 1917 or earlier (Flanders Wheel); 1984 (Asteroid).

Start position: As shown in the diagram.
Object: Move pieces along the lines only from circle to circle until the word FLANDERS can be read correctly around the rim of the wheel starting with the F at the top of the circle (now occupied by N).
Solution: 30 moves.
Rating: **

B10

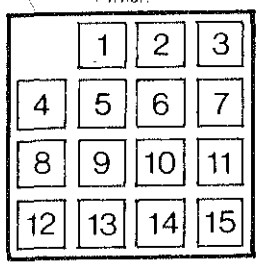
Start



Titles and Dates: **The '15' Puzzle**—the first proper sliding block puzzle to be produced on a commercial scale—invented in USA, most probably in 1878. Commercially produced and sold sometime in 1879 under the above title in USA and England. **Le Jeu du Taquin** (1879 or early 1880) was the same puzzle in France. **Boss puzzle** (1879) or early 1880) was the name for the German version (see Plate IV). **'14–15'** (possibly 1880 or 1881)—a later version proposed by Sam Loyd. Note that the '14–15' Puzzle differed from the '15' Puzzle—see below.
Start position: Random (15, Taquin, Boss)
As shown in the diagram (14–15)
Object: Finish as shown in the diagram except with pieces 14 and 15 reversed.
Solution: Possible for exactly 50% of all random start positions (15, Taquin, Boss). Impossible (14–15).
Comment: Two 'alternative' solutions were soon suggested (1881) for the '14–15' Puzzle. These are given as puzzles B11, B12.

B11

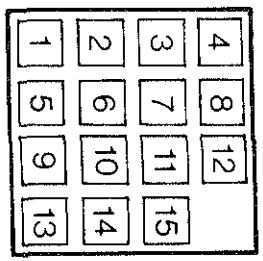
Finish



Start position: As shown for B10.
Object: Finish as shown in the diagram.
Solution: 44 moves.
Rating: * (*** in minimum moves).

B12

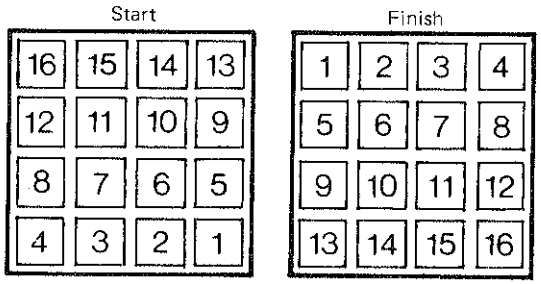
Finish



Start position: As shown for B10.
Object: Finish as shown in the diagram.
Solution: 39 moves.
Rating: * (*** in minimum moves).

B13

Title: **Tit-Bits Teaser No. 6.**
Date: *ca.* 1932.
Start position: As shown in the diagram—remove piece No. 16.
Object: Finish as shown in the diagram.



Solution: 78 moves (removing and replacing piece No. 16 not counted as moves).

Rating: * (** in minimum moves).

B14

Start position: As shown for B10.

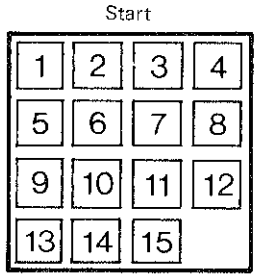
Object: Make a magic square (all orthogonals and the two main diagonals should add up to the same total).

Solution: 36 moves. The magic square to be found is given in the solutions section.

Comment: This puzzle was also suggested by H. E. Dudeney in his book *Puzzles and curious problems*.

Rating: *** (**** in minimum moves).

B15



Title: **The Spanish Dungeon**—from *Amusements in mathematics* by H. E. Dudeney.

Date: 1917 or earlier.

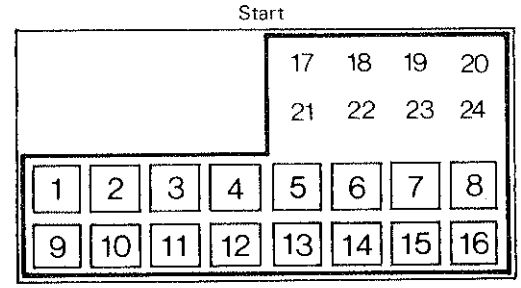
Start position: As shown in the diagram.

Object: Make a magic square (all orthogonals and the two main diagonals should add up to the same total).

Solution: 35 moves. The magic square to be found is given in the solutions section.

Rating: *** (***** in minimum moves).

B16



Title: **The Siberian Dungeons**—from *Amusements in mathematics* by H. E. Dudeney.

Date: 1917 or earlier.

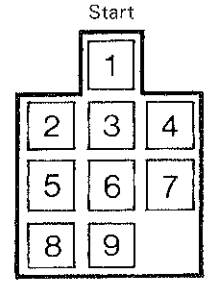
Start position: As shown in the diagram.

Object: Make a magic square (all orthogonals and the two main diagonals should add up to the same total).

Solution: 14 moves. Note that pieces can move any distance.

Rating: *** (**** in minimum moves).

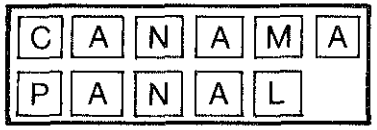
B17



Title: **The Fifteen Puzzle**—from *Wit sharpeners* (Anon.), a booklet published by the Padiham Advertiser, Burnley, England.
Date: 1944.
Start position: As shown in the diagram.
Object: Make a magic square (all orthogonals and the two main diagonals should add up to 15).
Solution: 24 moves.
Rating: * (** in minimum moves).

B18

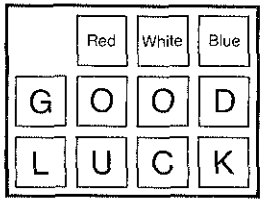
Start



Title: **Panama Canal Puzzle**—origin unknown.
Date: Unknown.
Start position: As shown in the diagram.
Object: Make the words read PANAMA CANAL.
Solution: 23 moves.
Rating: * (** in minimum moves).

B19, B20

Start

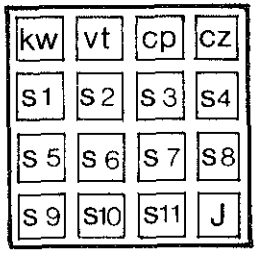


Title: **Good Luck**—origin unknown.
Date: 1919.
Start positions: As shown in the diagram for both puzzles.

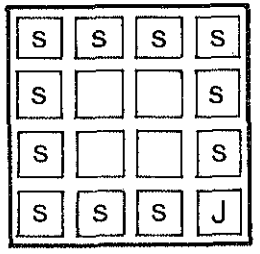
Object: B19—move Red, White and Blue into the centre row in the same order, so that the top row reads GOOD.
B20—from the same original start position, move Red, White and Blue into the bottom row in the same order, while the top row should read GOOD and the centre row LUCK.
Solutions: B19—28 moves
B20—27 moves.
Ratings: * (** minimum moves).

B21

Start



Finish

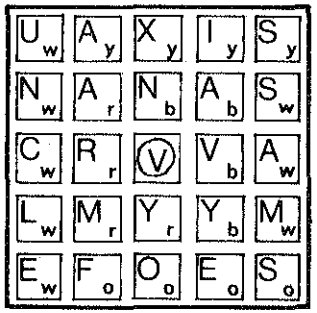


Titles: **Capturing the Kaiser**—USA; **Tit-Bits New Game.**
Date: 1914–18 (Kaiser); 1930s (Tit-Bits)
Pieces: KW = Kaiser Wilhelm; VT = Von Turpitz; CP = Crown Prince; CZ = Count Zeppelin; S = soldier; J = joker.
Start position: As shown in the diagram. Remove J (joker).
Object: Move to surround KW, VT, CP, CZ with soldiers, then replace joker at bottom right corner—see diagram.
Solution: 26 moves. Removing and replacing Joker do not count as moves.
Rating: * (** in minimum moves).

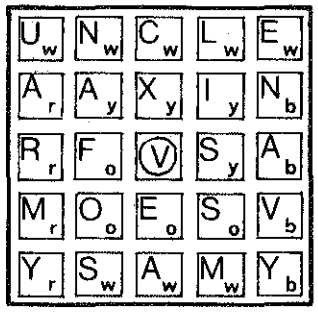
B22

Title: **Strategy**—submitted to *Popular Science Magazine*, USA, by Arthur L. Smith.
Date: September 1942.

Start



Finish



Pieces: AXIS—yellow (y); FOES—orange (o); UNCLE SAM—white (w); ARMY—red (r); NAVY—blue (b).

Start position: As shown—AXIS, FOES hemming in UNCLE SAM, ARMY, NAVY. Remove central Ⓥ Victory) block.

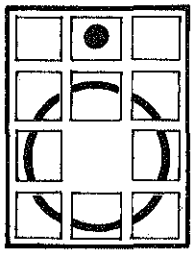
Object: Finish as shown in the diagram. UNCLE SAM, ARMY, NAVY surrounding AXIS, FOES. Replace Ⓥ in centre.

Solution: 116 moves. Removing and replacing Ⓥ do not count as moves.

Rating: ** (*** in minimum moves).

B23

Start



Titles: **Bull's Eye**—from *100 puzzles: how to make and how to solve them* by A. Filipiak; **Target; ZOT;** many others.

Date: 1942 (Filipiak).

Start position: As shown in the diagram.

Object: Move bull's eye (dot) inside target (large circle).

Solution: 17 moves.

Comment: There are many versions of this puzzle in many different forms—some have a 4 × 4 format—but they are all the same puzzle with the same solution. It is reckoned that many of the first copies of this puzzles were imitations of puzzle C2, 'Get my goat'. Manufacturers either did not realize that there was a rectangular block (it is difficult to see when the pieces are close together in the box)—or they did not appreciate the 'secret' and were unable to solve it. Later imitators were obviously 'copying copies'.

Rating: * (** in minimum moves).

B24

Title: **ZOX.**

Date: 1970s.

Start position: As in the previous puzzle (B23) except that the two blank pieces are Z (top left) and X (top right).

Object: Move the dot (top centre) inside the completed circle whilst the pieces Z and X must be in their original positions.

Solution: 31 moves.

Comment: Is it possible?—lateral thinking will be required to solve this one!

Rating: ** (*** in minimum moves).

B25

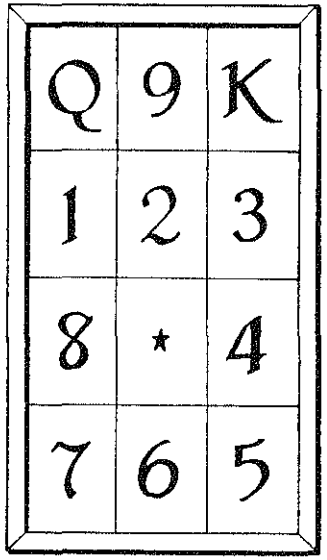
An alternative version of the previous puzzle (B24), with an added twist.

Title: None—invented by N. Takashima, Japan and published in the *Academy of Recreational Mathematics Magazine*, Japan.

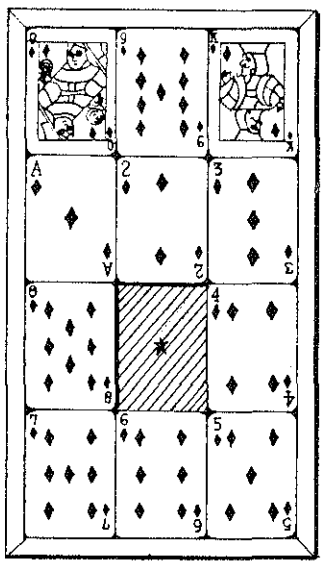
Date: March 1985.

Start position: As shown in the diagram. Note that the base of the tray is marked as in the first diagram.

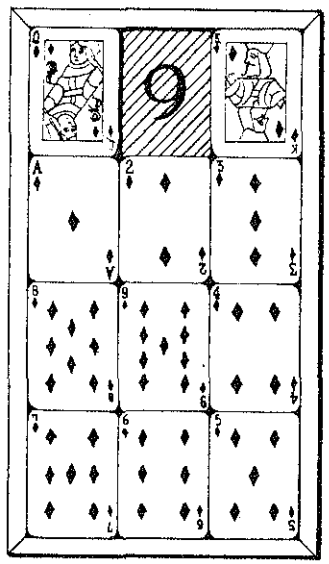
Base of tray



Start



Finish



Object: Move the nine of diamonds into the vacant space, leaving all other cards in their original positions (as in the last diagram).

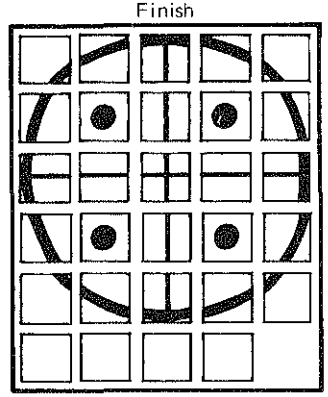
Solution: 47 moves.

Comment: More lateral thinking will be required to solve this one. Why have diamonds been chosen, and why would it be better to use the ten of diamonds rather than the seven?

Rating: ** (*** minimum moves).

B26

Finish



Title: **The Quadrant Puzzle**—manufactured by Decon, USA.

Date: 1970.

Start position: As shown in the diagram—except that the four dots start at the bottom where the four blank pieces are.

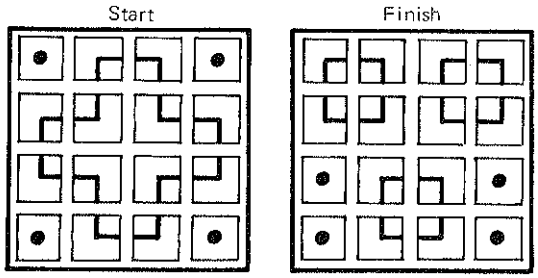
Object: Finish as shown in the diagram.

Solution: 74 moves.

Rating: * (** in minimum moves).

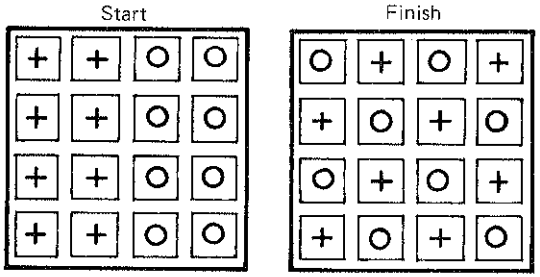
B27

Title: **Wreck-Reation**—manufactured by Peterson Games, California, USA.



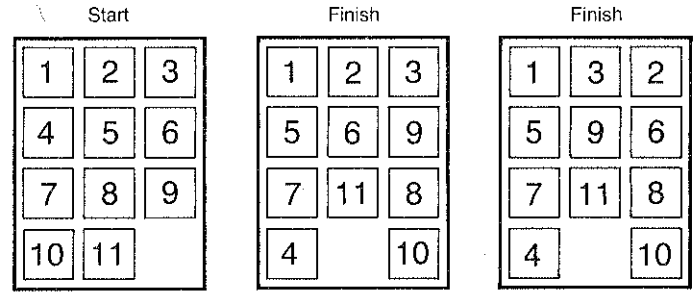
Date: 1975.
Start position: As shown in the diagram—remove any corner piece.
Object: Finish as shown in the diagram—replace piece removed.
Solution: 18 moves. Removing and replacing piece do not count as moves.
Rating: * (*** in minimum moves).

B28



Title: **The Dormitory Dash**—manufactured by Peterson Games, California, USA.
Date: 1975.
Start position: As shown in the diagram—remove any one piece.
Object: Finish as shown in the diagram—replace piece removed.
Solution: 25 moves. Removing and replacing piece do not count as moves.
Rating: * (*** in minimum moves).

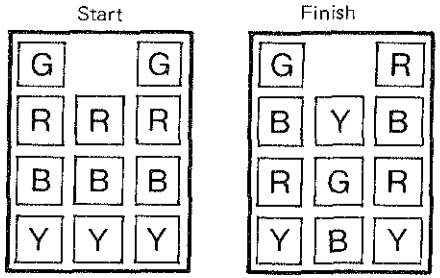
B29, B30



Title: **Vanish Mystery Puzzle**—manufactured by Gordon Bros, USA (see Plate III).
Date: 1979.
Pieces: This puzzle is not only a sliding block puzzle but a 'vanish illusion' at the same time. When the pieces are arranged in the start position, pictures on them show six magicians (see Plate III). When in one Finish position (B29) there are five magicians and a pair of shoes, and in the other Finish position (B30) there are just five magicians. Where does the sixth magician disappear to?
Start position: As shown in the diagram for both puzzles.
Object: B29—move to the Finish position shown in the diagram.
B30—move to the Finish position shown in the diagram.
Solutions: B29—15 moves
B30—27 moves.
Ratings: *

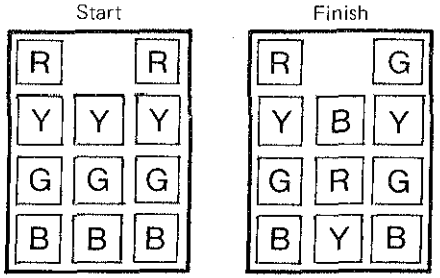
B31

Title: **Parking Lot Parable**—produced by Products of the Behavioral Sciences Inc. USA.
Date: 1971.
Pieces: G = green; R = red; B = blue; Y = yellow.
Start position: As shown in the diagram.



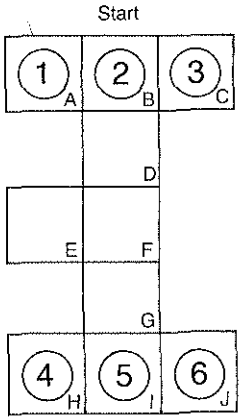
Object: Finish as shown in the diagram—no two pieces of the same colour touching.
Solution: 20 moves.
Rating: * (** in minimum moves).

B32



Title: **Shifty**—manufactured by Gordon Bros, California, USA.
Date: 1974.
Pieces: R = red; Y = yellow; G = green; B = blue.
Start position: As shown in the diagram.
Object: Finish as shown in the diagram—no two pieces of the same colour touching. There are fifteen possible different patterns.
Solution: 20 moves. One other pattern also requires 20 moves, six require 24 moves, while one requires 36 moves (the longest).
Rating: * (** in minimum moves).

B33



Titles: **A Motor-Car Problem**—submitted to *Strand Magazine* probably by H. E. Dudeney. **Germans vs. Allies Puzzle**—sold as a card with 'push out' pieces.
Date: 1903 (Motor-Car); 1918 (Germans).
Start position: As shown in the diagram.
Object: Exchange pieces 1, 2, 3 with 4, 5, 6. Pieces may be in any order. Note that a piece may move any distance in a move.
Solution: 17 moves.
Rating: *

B34

Titles: **Peyo Switchit**—manufactured by Intermundus, Holland (see Plate III); **Logi Toli**—manufactured in Hungary but also from a book *Početní a jiné hříčky a zàbarg* by P. Lešan (Czechoslovakia).
Dates: 1927 ('Početní. . .'); 1976 (Peyo); 1978 (USA Patent No 4 097 049 by M. F. DeVos, issued for the Peyo design); *ca.* 1981 (Logi Toli).
Pieces: Peyo Switchit is manufactured with tongued and grooved pieces, which will not fall out. It is a double-sided puzzle with pictures on both sides: when it is

solved on one side it automatically provides the start position for the other side.

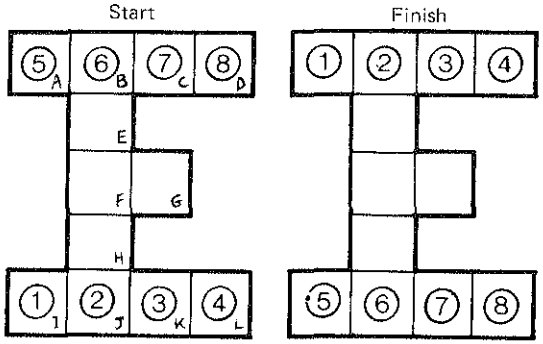
Start position: As shown in the diagram for the previous puzzle B33.

Object: Exchange pieces 1 with 4, 2 with 5 and 3 with 6. Note that a piece may move any distance in a move.

Solution: 22 moves.

Rating: *

B35



Titles: **A Motor Garage Puzzle**—submitted to *Strand Magazine* No. 233 by H. E. Dudeney and later published in his book *Amusements in mathematics*; **Parka Car**—made by Strataboard Ltd., (see Plate III); **The Sputnik Puzzle**—one of a series of puzzles under the general title of 'Magnetic spaceman'; also one of another series under the general title of **Tricky Button Puzzles** made in Hong Kong.

Dates: May 1910 (*Strand Magazine*); 1976 (Parka Car); 1984 (Sputnik and Tricky Button).

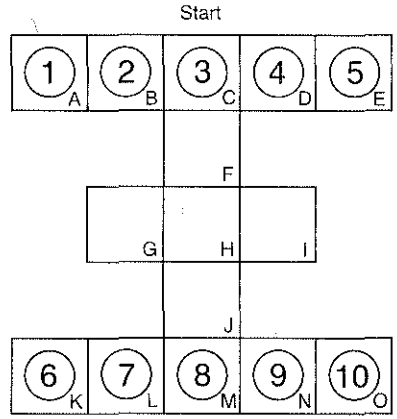
Start position: As shown in the diagram.

Object: Finish as shown in the diagram. Note that a piece can move any distance in one move.

Solution: 43 moves.

Rating: * (*** minimum moves).

B36



Title: **Zrcadlové E.**

Date: Unknown.

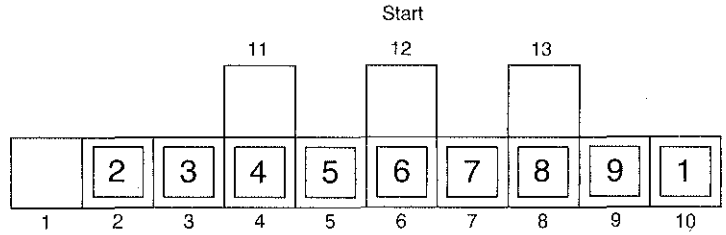
Start position: As shown in the diagram.

Object: Exchange pieces 1 with 6; 2 with 7 etc. Note that a piece may move any distance in a move.

Solution: 32 moves.

Rating: *

B37



Title: **Nine Men in a Trench**—from *Puzzles and curious problems* by H. E. Dudeney.

Date: 1917 or earlier.

Start position: As shown.

Object: Move piece 1 to position 1 at the left of the line—the

other pieces to be returned to their start positions. Note that a piece may move any distance in one move.

Solution: 28 moves.

Rating: *

B38

Title: None—one of a series of four under the general title of **Tricky Button Puzzles** made in Hong Kong.

Date: 1984.

Pieces: Dumbell shaped pieces are free to slide in a slot. The design is exactly the same as for the previous puzzle (B37). Four pieces are one colour, four another colour and one a third colour.

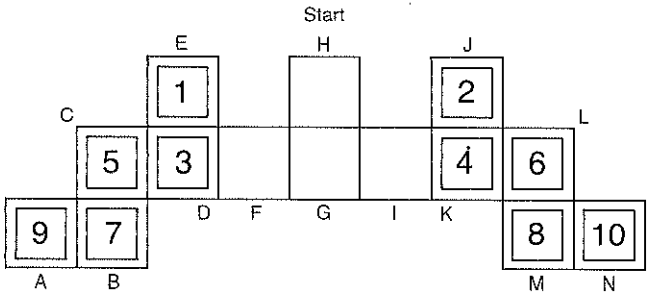
Start position: As shown in the diagram for the previous puzzle—four pieces of one colour should be at positions 2–5, four of the second colour at positions 6–9, and the single piece with a third colour at 10.

Object: Move piece 1 (at position 10) to the far left (position 1) and exchange the positions of the other coloured pieces (2–5 to exchange with 6–9). Note that a piece may move any distance in one move.

Solution: 57 moves.

Rating: * (** minimum moves).

B39



Title: None—submitted to *Woman's Home Companion*, USA, by S. Loyd.

Date: November 1904.

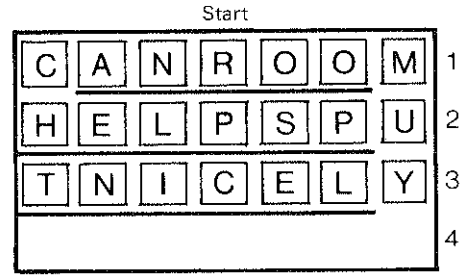
Start position: As shown in the diagram.

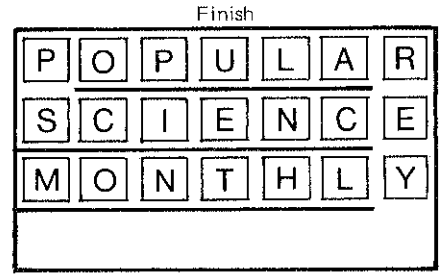
Object: Exchange the odd and even pieces—1 with 2, 3 with 4, etc. Note that pieces can move any distance in one move.

Solution: 71 moves.

Rating: ** (**** in minimum moves).

B40





Title: None—from *Popular Science Magazine*, USA.

Date: October 1933.

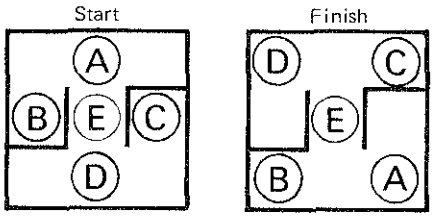
Start position: As shown in the diagram—(or any other at random).

Object: Finish as shown in the diagram. Note immovable partitions.

Solution: 68 moves.

Rating: *

B41



Titles: **The So-Easy**—manufactured by Feltham & Co., London (see Plate IV); **Le Jeu des Quatre Coins**—by Simonin-Cuny, France; **Aux Quatre Coins**—by N. K. Atlas, France.

Dates: 1885–1905 (all).

Pieces: A = yellow; B = blue; C = red; D = green; E = black.

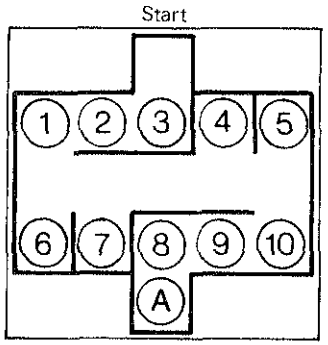
Start position: As shown in the diagram.

Object: Finish as shown in the diagram. Note the immovable partitions—pieces must not cross over them. Note also that pieces may move any distance in one move.

Solution: 17 moves.

Rating: *

B42



Titles: **The Teaser**—manufactured by Feltham & Co., London (UK Patent No. 3492) (see Plate IV); **L'Evasion Difficile**—by Jeux et Jouets Français, France.

Dates: 1885–1910 (both).

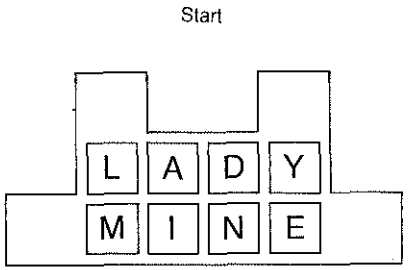
Start position: As shown in the diagram.

Object: Move A to the topmost position. Note the immovable partitions—pieces must not cross over them. Note also that pieces may move any distance in one move.

Solution: 18 moves.

Rating: *

B43



Title: **Anagram Blocks**—from *Sam Loyd and his puzzles* by S. Loyd.

Date: 1928.

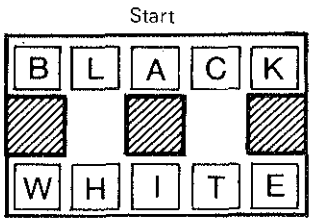
Start position: As shown in the diagram.

Object: Rearrange the letters so as to spell MAIDENLY—MAID in the top row and ENLY in the bottom row. Note that a piece may only move one space—to the next adjacent square—in each move.

Solution: 26 moves.

Rating: * (*** in minimum moves).

B44



Titles: **Black and White**—submitted to *Popular Mechanics Magazine* by K. Wells (see Plate IV). **Black and White Brain Game**—made by Toitoy (Ideal Brain Teaser Series). Also **Zebřík** (= ladder), from Czechoslovakia.

Dates: May 1971 (*Popular Mechanics*); 1970s (others).

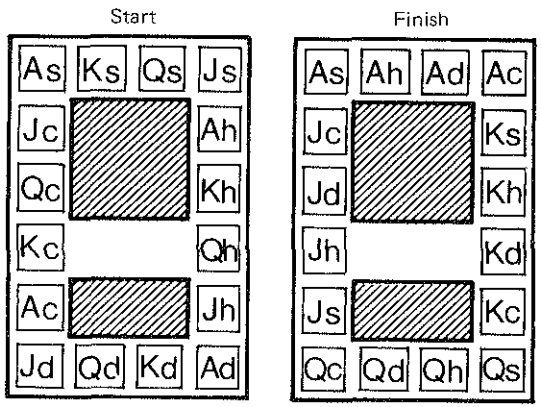
Start position: As shown in the diagram.

Object: Exchange positions of WHITE and BLACK. Note immovable blocks (shaded areas).

Solution: 62 moves (85).

Rating: * (*** in minimum moves).

B45



Title: **Contract Bridge Solitaire**—manufactured by H. W. Spaulding, USA.

Date: Patented in 1932 by H. W. Spaulding (USA Patent No. 1 879 571).

Pieces: Represent playing cards A = ace; K = king; Q = queen; J = jack; s = spades; h = hearts; d = diamonds; c = clubs.

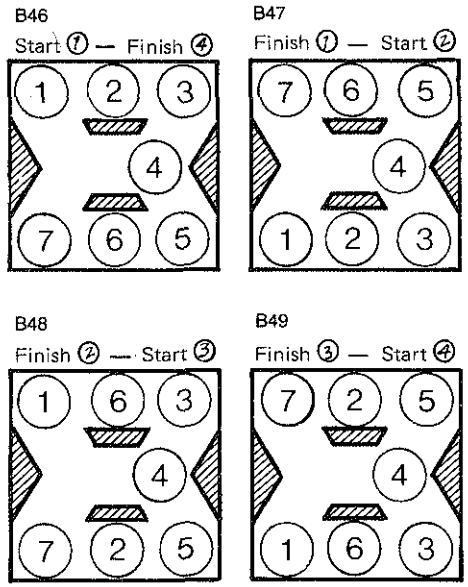
Start position: As shown in the diagram.

Object: Finish as shown in the diagram. Note immovable blocks (shaded areas).

Solution: The only thing needed to solve this puzzle is patience —so no solution is given.

Rating: *

B46–B49



Titles: **Tantalising 7; Nutty Numbers.**

Date: 1960s.

Start positions: As shown in the diagrams.

Objects: Finish as shown in the diagrams. Each finish position is the start for the next puzzle. Note immovable blocks (shaded areas).

Solutions: B46—35 moves
B47—54 moves
B48—35 moves
B49—54 moves.

Ratings: ** (*** in minimum moves).

B50–B59

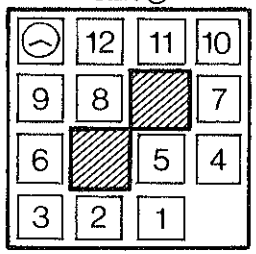
Title: **Time Puzzle**—manufactured by the Embossing Company, New York (see Plate III).

Date: *ca.* 1937.

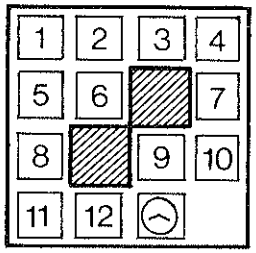
Start positions: As shown in the diagrams.

Object: Finish as shown in the diagrams. Each finish position

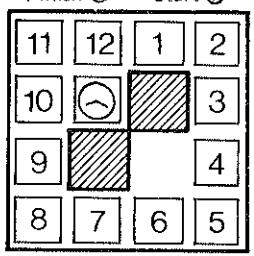
B50 Start ①



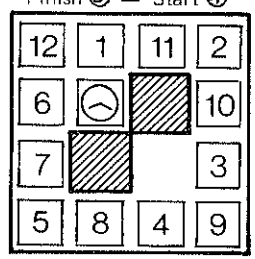
B51
Finish ⑦ — Start ②



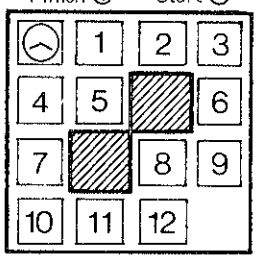
B52
Finish ② — Start ③



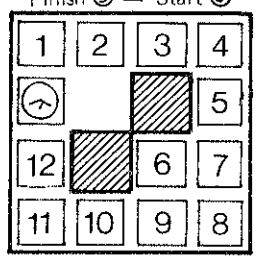
B53
Finish ③ — Start ④



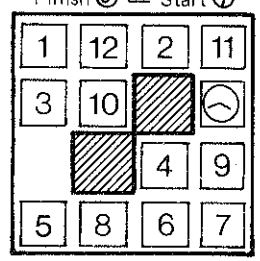
B54
Finish ④ — Start ⑤



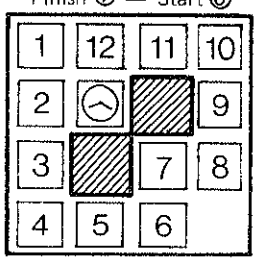
B55
Finish ⑤ — Start ⑥



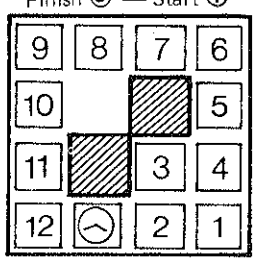
B56
Finish ⑥ — Start ⑦



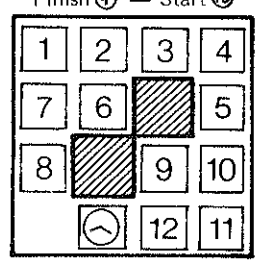
B57
Finish ⑦ — Start ⑧



B58
Finish ⑧ — Start ⑨



B59
Finish ⑨ — Start ⑩

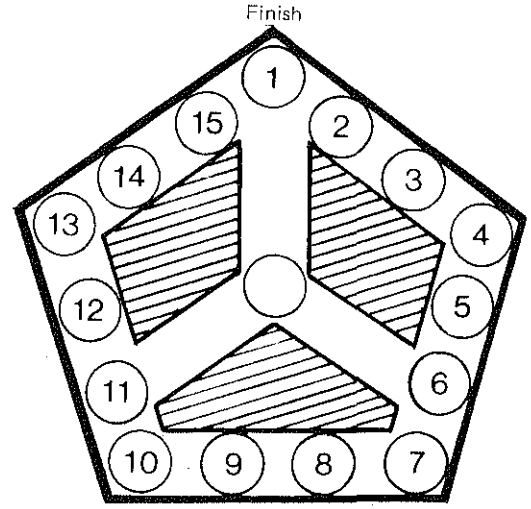


is the start for the next puzzle. Note immovable blocks (shaded areas).

Solution: B50–178 moves. B51–59—no solutions given.

Ratings: * (** in minimum moves).

B60



Title: **The New 15 Puzzle** (see Plate IV).

Date: 1923.

Start position: As shown in the diagram—except that piece 15 should be in the centre.

Object: Finish as shown in the diagram. Pieces may only be slid from circle to circle.

Solution: 81 moves.

Comment: Other puzzles that are given are to place the pieces on the outer circle so that each and every side (of four numbers) totals 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, or 37.

Rating: **** (***** in minimum moves).

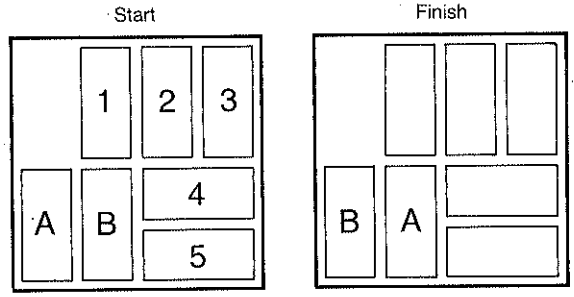
5

SLIDING BLOCK PUZZLES WITH RECTANGULAR PIECES, C1–C76

The invention of the rectangular piece added a new dimension to sliding block puzzles, and puzzles could be designed to be hard merely by the shape of the pieces. In the main they are much more interesting as puzzles than the previous ones and some of the best designs are in this category. Usually the problem is just to solve the puzzle; experts may prefer to see if they can find the minimum move solutions. For an explanation of what constitutes a move, refer to the 'moves' section in Chapter 1 (see page 5).

The better puzzles are: C1, C2, C3, C4, C14, C19, C27, C29, C41, C42, C45, C46, C47, C52, C57, C67, C70, C71, C73, C75, C76.

C1



Titles and dates: Patented in 1894 by H. Walton (USA Patent No. 516 035); also in *Games and puzzles for addicts* by R. Millington (1979).

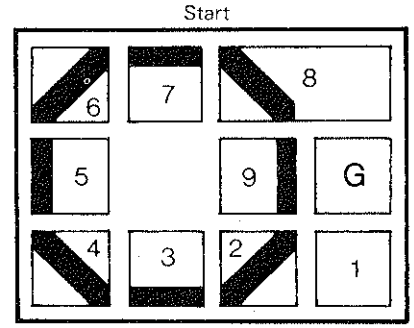
Start position: As shown in the diagram.

Object: Exchange positions of pieces A and B—other pieces to be arranged as in the diagram.

Solution: 56 moves.

Rating: **

C2



Titles and Dates: **Get My Goat**—patented in USA in 1914 (No. 1 112 746) by J. I. Wiley (see Plate VI); **Kapture The Kron Prinz**—patented in UK in 1916 (No. 110 186) by R. W. Hunter and J. C. Wilson (see Plate VI); **The Boogie Man**—manufactured by Sherms, Connecticutt, USA; **Put Hitler in the Doghouse** (1942). A clever modern version is the **Helio** puzzle designed by N. Yoshigahara of Tokyo, Japan (1983) and manufactured by Hanayama, Japan.

Start position: As shown in the diagram.

Object: Move G (the goat) inside the fence.

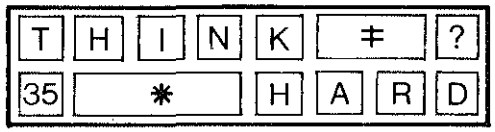
Solution: 28 moves.

Rating: ***

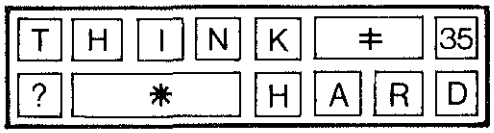
Comment: In this puzzle there is just one rectangular piece and at first sight it makes the puzzle impossible. Many later copies of this puzzle failed to appreciate the simple but unexpected solution and divided the rectangular piece into two squares, thus destroying the whole point of the puzzle and making it very simple to solve (*cf.* B23).

C3

Start



Finish



Title: **Qwik-Sane**—by J. R. O'Neil, manufactured by Wiff N' Proof Inc., USA (see Plate V).

Date: 1960s.

Start position: As shown in the diagram—remove piece 35.

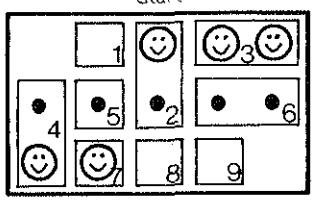
Object: Finish as shown—replace piece 35.

Solution: 35 moves. Removing piece 35 does not count as a move, but replacing it at the end does. The definition of a move for this puzzle is: any number of pieces together as a group in any *one* direction.

Rating: ***

C4

Start



Titles: **Line Up the Quinties** by R. W. Fatiguant, USA; **The Line Change,** Canada. This puzzle was originally produced to celebrate the birth of the famous Dionne quintuplets.

Dates: 1934 (Quinties); 1981 (Line Change).

Start position: As shown in the diagram.

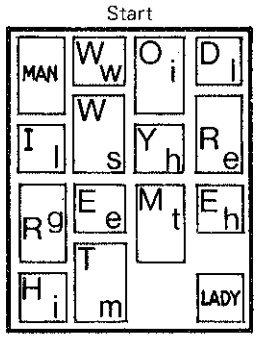
Object: Move the faces to form a line where the dots are.

Solution: 30 moves (33).

Rating: *

C5

Start



Title: **The Merry Widow**.

Date: Patented in 1909 (US Patent No. 922 002) by L. C. Koehler and W. A. Butler.

Start position: As shown in the diagram—small letters spell out 'will she get him'.

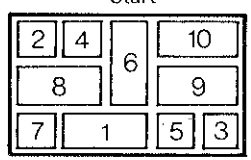
Object: Move M (MAN) down to L (LADY) at the bottom right (on her left). Arrange the other pieces to spell THE MERRY WIDOW (with the capitals).

Solution: 89 moves.

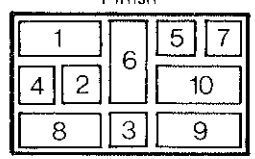
Rating: **

C6

Start



Finish

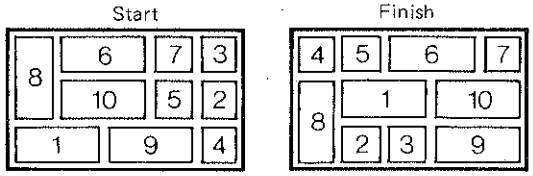


Title: **A Yank Thru the Lines**—one of a series of three under the general title of 'Slidem Solitaire Puzzles' produced by Electric Corporation of America, Chicago.

Date: 1942.

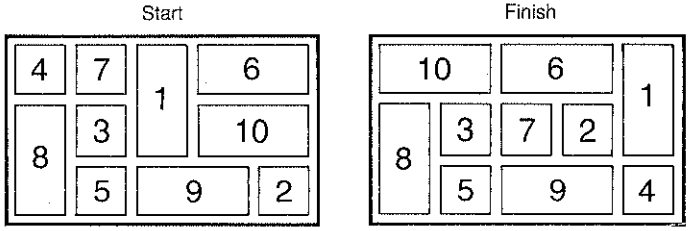
Pieces: Have pictures of soldiers with rifles on them.
Start position: As shown in the diagram. Remove piece 1.
Object: Reach finish position shown in the diagram. Move piece 7 (Yank) to top right corner. Other pieces form a picture. Replace piece 1 at the end.
Solution: 13 moves (17). Removing and replacing piece 1 do not count as moves.
Rating: *

C7



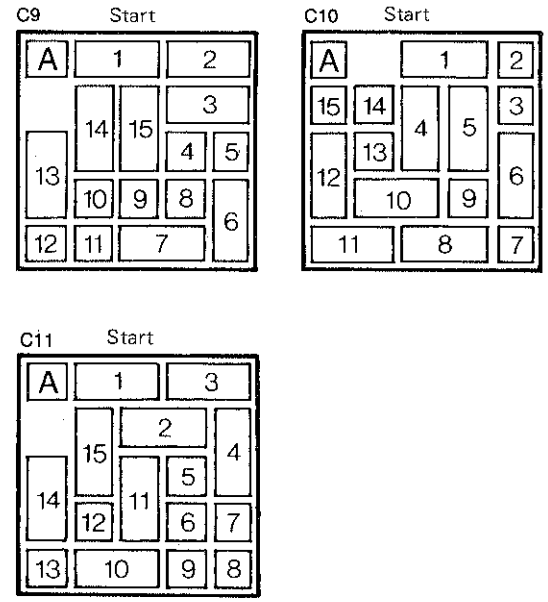
Title: **Bombing Tokyo**—one of a series of three under the general title of 'Slidem Solitaire Puzzles' produced by Electric Corporation of America, Chicago.
Date: 1942.
Pieces: Have pictures of aeroplanes and bombing on them.
Start position: As shown in the diagram. Remove piece 9.
Object: Reach finish position shown in the diagram. Move piece 4 (aeroplane) to top left corner. Other pieces form a picture. Replace piece 9 at end.
Solution: 20 moves (23). Removing and replacing piece 9 do not count as moves.
Rating: *

C8



Title: **Tank Attack**—one of a series of three under the general title of 'Slidem Solitaire Puzzles' produced by the Electric Corporation of America, Chicago.
Date: 1942.
Pieces: Have pictures of tank and other artillery on them.
Start position: As shown in the diagram. Remove piece 1.
Object: Reach finish position shown in the diagram. Move piece 4 (aeroplane) to top left corner. Other pieces form a picture. Replace piece 9 at end.
Solution: 19 moves (22). Removing and replacing piece 1 do not count as moves.
Rating: *

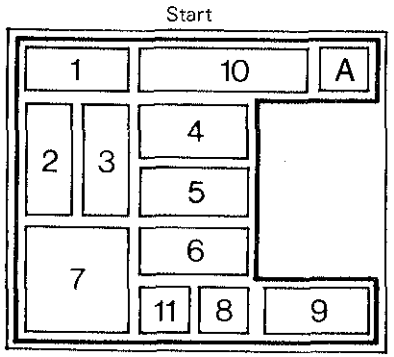
C9–C11



Title: **Pazienza** (Patience)—Italy.
Date: 1978.
Start positions: As shown in the diagrams.
Object: C9, C10, C11—move A to bottom right corner.

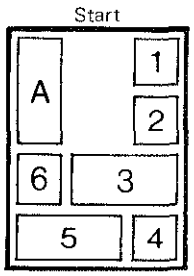
Solutions: C9—60 moves
C10—54 moves
C11—51 moves.
Ratings: *

C12



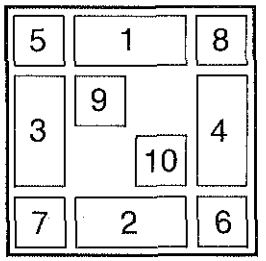
Title: **The Infants' Hospital Puzzle**—produced to aid the Infants' Hospital, Westminster, London, and manufactured by Chad Valley Co. Ltd., (see Plate VI).
Date: 1920.
Pieces: Each piece has a picture on it—piece A is the child.
Start position: As shown in the diagram. Remove pieces 10 and 11.
Object: Move A to bottom right corner.
Solution: 28 moves (35).
Rating: *

C13



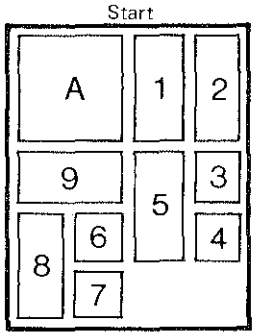
Title: None—submitted to *Amateur Handicraft Magazine*, USA, by F. Clark Hughes.
Date: 1947.
Start position: As shown in the diagram.
Object: Move A to bottom right corner.
Solution: 23 moves (27).
Rating: *

C14



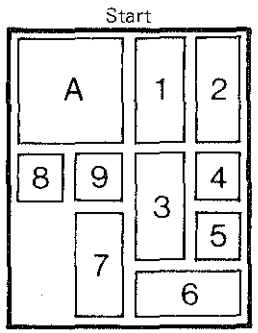
Title: Unknown.
Date: 1901—US Patent No. 668 386, by Frank E. Moss. The first puzzle design (was it ever produced?) to include square and rectangular pieces with full movement of all pieces.
Pieces: In the patent the edges of the box or tray are to be coloured—one colour for each of the four edges. The pieces should be similarly coloured with a fifth colour for the two pieces in the centre.
Start position: As shown in the diagram—pieces should be on the opposite side of the tray to their respective colours.
Object: Exchange pieces 1 with 2, 3 with 4, 5 with 6, 7 with 8 so that the colours on the edge of the tray match the colours on the adjacent pieces.
Solution: 70 moves (76).
Rating: **

C15



Title and date: Unknown.
Pieces: Original pieces had Spanish names—A = Reo; 1 = Fiscal; 2 = Juez; 3, 4, 5, 6, 7 = Jaula; 8 = Gestor; 9 = Alcalde.
Start position: As shown in the diagram.
Object: Move A to bottom centre.
Solution: 63 moves (70).
Rating: **

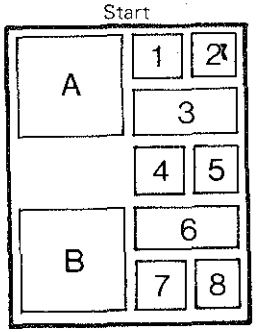
C16



Title and date: Unknown.
Start position: As shown in the diagram.

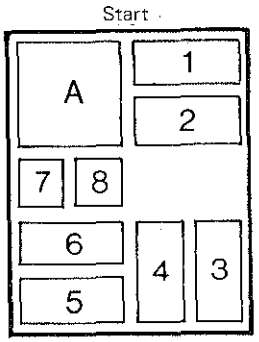
Object: Move A to bottom left corner.
Solution: 40 moves (45).
Rating: **

C17



Titles: **10 Block Puzzle**—from *100 puzzles: how to make and how to solve them* by A. Filipiak; **Traffic Cop Tangle.**
Date: 1942 (Filipiak).
Start position: As shown in the diagram.
Object: Exchange the positions of pieces A and B.
Solution: 47 moves (55).
Rating: *

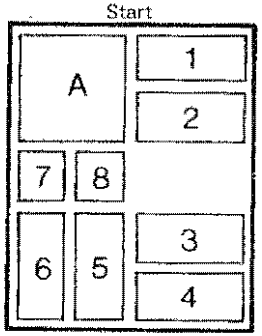
C18



Title: None—manufactured by Galt Toys.
Date: 1970s.

Start position: As shown in the diagram.
Object: Move A to bottom right corner.
Solution: 33 moves (34).
Rating: *

C19



Titles and dates: According to Gardner (1975) the earliest known version was called the **Penant Puzzle**. It was copyrighted in 1909 by L. W. Hardy (*cf.* C43) and was manufactured by the O. K. Novelty Co. in Chicago, USA. Further USA versions were copyrighted in 1926 by J. W. Hayward and in 1927 by F. E. Aaron. A UK version was patented (Patent No. 381 813) in 1932 by I. Isowitsky. This puzzle has probably been the most widely sold after the '15' Puzzle (B10). A few of the countless versions that have appeared include: **Dad's Puzzler** (1926) (see Plate VI); **Tit-Bits Teaser No. 1** (1927); **Humdinger Puzzler; The Moving Puzzle** (1927); **Comet.**

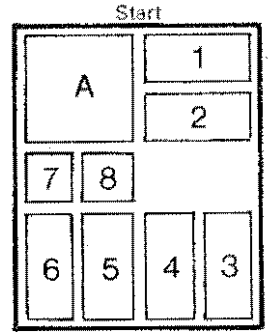
Pieces: One version has the words A = my photo; 1 = stand; 2 = Dear sir; 3 = those; 4 = me; 5 = can; 6 = you; 7 = now; 8 = awe.

Start position: As shown in the diagram.

Object: Move A to bottom left corner. Additionally, in the puzzle with words on the pieces, it is required to make a sentence out of the words.

Solution: 59 moves (62).
Rating: **

C20, C21



Title: **Jusso – The Nine Block Puzzle**—manufactured by Kum-Bak Sports, Toys & Games Ltd., London. The instructions also give details of puzzle C19.

Date: 1930s.

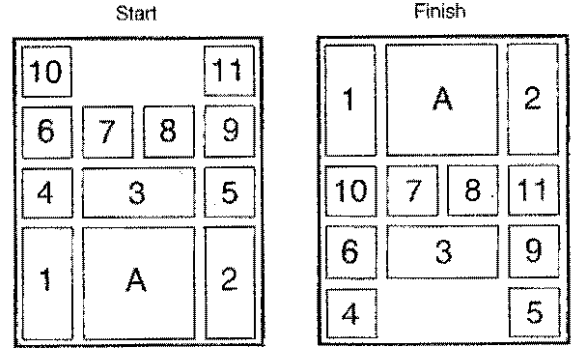
Start positions: As shown in the diagram for both puzzles.

Object: C20—move A to bottom left corner
C21—move A to top right corner.

Solutions: C20—45 moves (47).
C21—26 moves (27).

Ratings: *

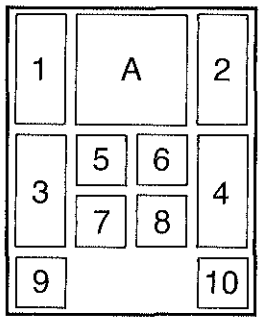
C22



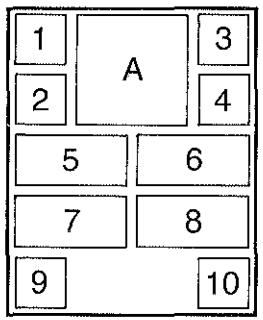
Title: **Mickey and Minnie Compact Puzzle**—manufactured by Tomy, Japan (see Plate V).
Date: 1984.
Pieces: Make a picture which has to be altered so that pieces 1, A, 2 are at the top.
Start position: As shown in the diagram.
Object: As shown in the diagram.
Solution: 77 moves (90).
Rating: **

C23–C26

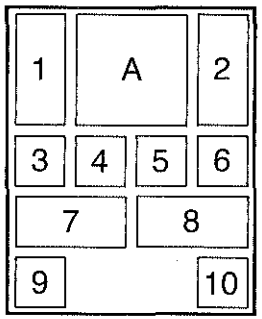
C23 Start



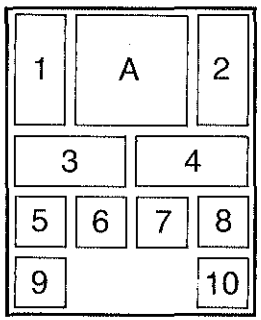
C24 Start



C25 Start



C26 Start

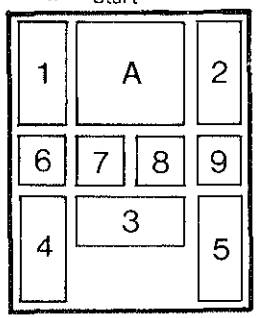


Title: **Royal Out Game** (1 set 5 challenge)—devised by Dr Kobayashi, Professor at Kagawa University, Japan, and manufactured by Hanabishi, Japan.

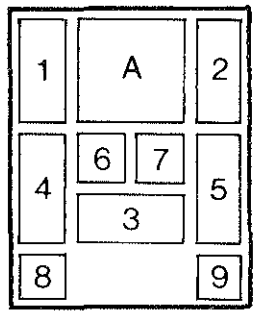
Date: 1982.
Start positions: As shown in the diagrams.
Object: Move A to bottom centre.
Solutions: C23—28 moves (34)
C24—32 moves (35)
C25—40 moves (51)
C26—49 moves (64).
Comment: There are five puzzles given in the instructions, the fifth one being the same as C27(d).
Ratings: *

C27a–d

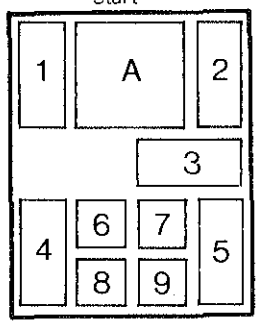
a Start



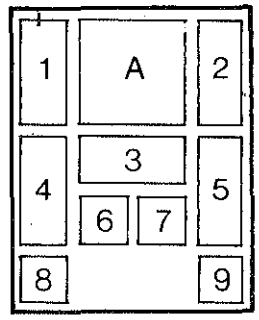
b Start



c Start



d Start



Titles: **L'Ane Rouge; Psychotease; Drop-Out; Intrigue; Mov-It; Hako; Ego Buster; Mariner; Simba; Moon Hunt;** etc.—from various countries. Probably the third most sold puzzle after the '15' puzzle (B10) and C19.

Dates: Patented in UK in 1932 (No. 411 515) by J. H. Fleming. Both this patent and a later UK Patent (No. 592 536) in 1946 by H. B. Saunders provides for a slit at the bottom end of the puzzle (next to the vacant space) allowing piece A only—which may be thinner than the others—to slide out of the box. From this comes the name 'Drop Out'.

Variations: An extremely well-made version of this puzzle made of superior quality plastic appeared on the market in the early 1980s. Made in Switzerland by Verlag Kings Games AG and called **Relax**, it has magnets in all the pieces, a magnetized board, and a transparent case. It has an unusual feature in that piece A is really two pieces—a small square being set into the large square along its bottom edge. When piece A reaches its final destination the small square can be slipped from piece A into a special space in the bottom edge of the tray (see Plate V).

An interesting version has recently been produced by Apollo-Sha Co., Japan (1983). Piece A has a transparent cylinder fixed on top of it inside which is a magnetized disc with a picture of a broken heart. When piece A reaches its final solution position a magnet with reverse polarity in the base of the tray makes the broken heart flip over. The picture is now a complete heart! The other pieces have pictures of Micky Mouse and Minnie Mouse, etc. (see Plate V).

In a further version produced by Kelrack Ltd., England, piece A has been made into a cage, inside which there are 'Mintman Puzzle Mints'. As all the pieces are tongued and grooved it is impossible to remove any of the pieces from the box. The only way to get at the mints is to solve the puzzle. Piece A will then protrude sufficiently far out of a hole in the bottom end of the box for the mints to be extracted from underneath (see Plate V).

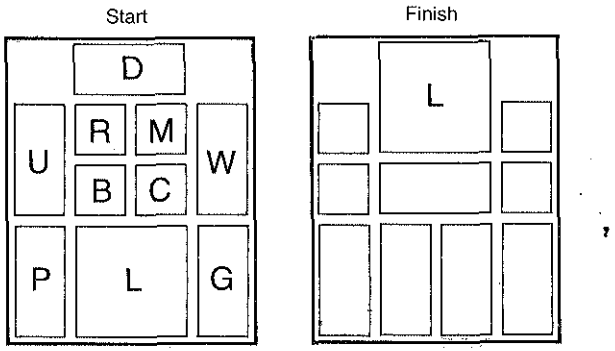
Start positions: As shown in the diagrams—four different.

Object: Move A to bottom centre. Some puzzles had a slot at the bottom centre to remove piece A when it got there. Despite the different start positions it is basically one puzzle.

Solutions: (a) 60 moves (67)
(b) 70 moves (81)
(c) 78 moves (89)
(d) 81 moves (90).

Ratings: **

C28



Titles: **Let Me Through; Traffic Jam** by Shafir Games, Israel.

Date: 1981.

Pieces: Represent people in a bus (Let Me Through). L = large lady; P = lady with purse; G = man with guitar; U = man with umbrella; W = workman with tools; D = dog; R = girl with ribbon; M = man with moustache; B = baby; C = baby carriage.

Start position: As shown in the diagram.

Object: To move L (the large lady) to the other side of the bus (top centre) and arrange other pieces as shown in the diagram.

Solution: 61 moves.

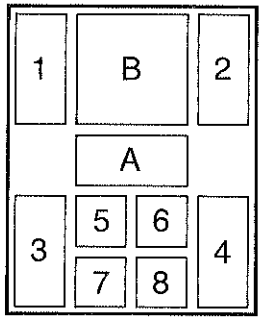
Rating: **

C29

Titles: **The Home Team Football Puzzle**—manufactured by Lott's Bricks. Also **Puffin Puzzle** by Puffin Toys.

Dates: 1940s (Home team football); 1981 (Puffin).

Start



Pieces: 'Home Team Football' pieces represent players, with piece A being the ball. In the centre at either end are two football goals. Piece B is the 'Home team'.
'Puffin' pieces A and B are the body and head respectively of a puffin. The tray has the feet of a puffin at the centre of the bottom edge.

Start position: As shown in the diagram.

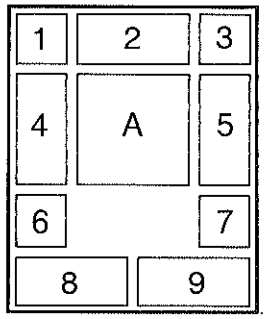
Object: Put the football in front of the goal with the Home Team immediately behind, or join the Puffin's head and body to its feet. In other words, move A to the bottom centre with B immediately above it, also in the centre.

Solution: 54 moves (64).

Rating: **

C30

Start



Title: **Top Secret**—origin unknown.

Date: 1940s or 1950s.

Start position: As shown in the diagram.

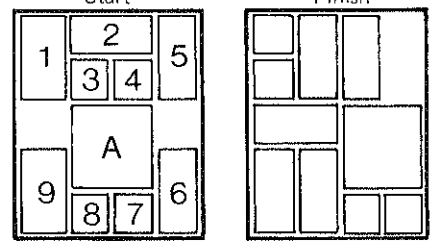
Object: Move A to bottom centre.

Solution: 67 moves (71).

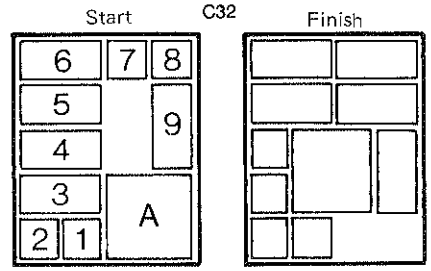
Rating: **

C31–C40

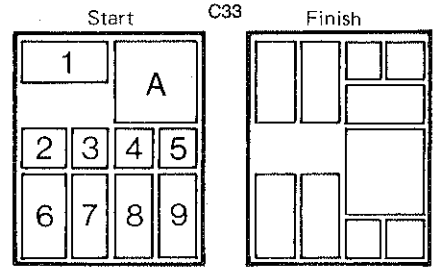
Start C31 Finish



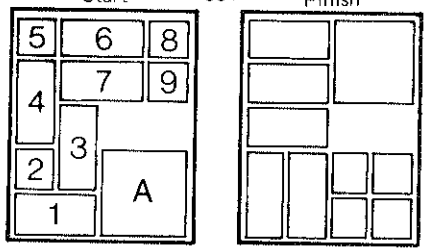
Start C32 Finish



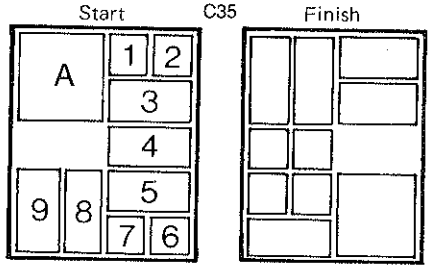
Start C33 Finish



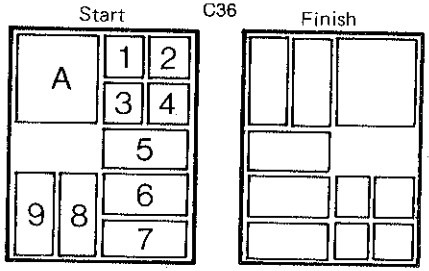
Start C34 Finish



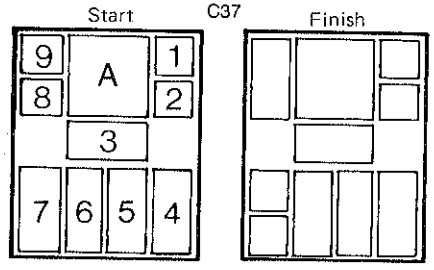
Start C35 Finish



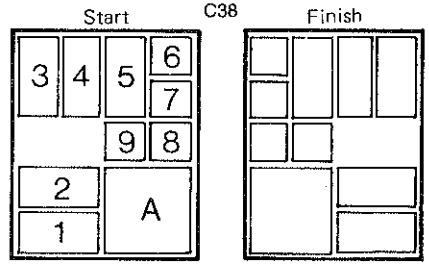
Start C36 Finish



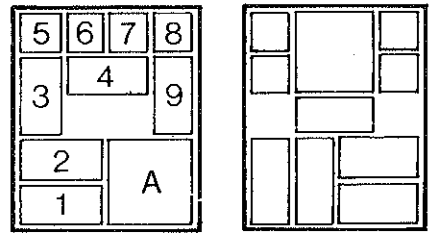
Start C37 Finish



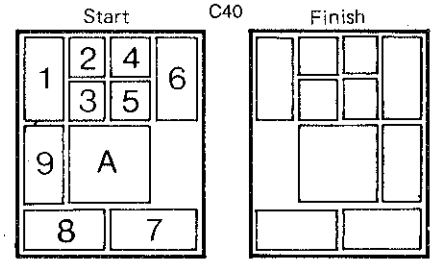
Start C38 Finish



Start C39 Finish



Start C40 Finish



Title: **Khum Pan**—after an old Thai hero. This puzzle, manufactured in Thailand, set the ten puzzles illustrated plus the one shown as C27.

Date: 1970s.

Start positions: As shown in the diagrams—to the left of each pair of diagrams.

Object: Move to position indicated to the right of each pair of diagrams.

Solutions:

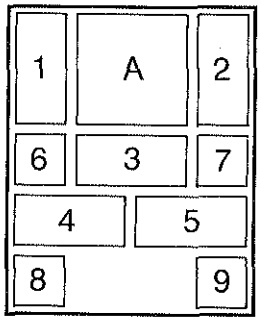
C31—55 moves (64)	C34—39 moves (44)
C32—25 moves (30)	C35—101 moves (110)
C33—20 moves (24)	C36—28 moves (34)

C37—40 moves (44)
C38—21 moves (23)
C39—53 moves (57)
C40—21 moves (21).

Ratings: * to **

C41

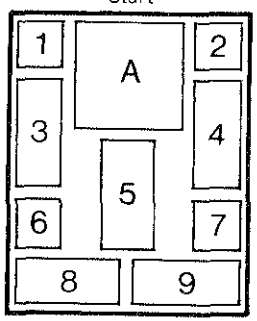
Start



Title: **Ushi Puzzle**—made by Himawari, Japan.
Date: 1981.
Start position: As shown in the diagram.
Object: Move A to bottom centre.
Solution: 98 moves (108).
Rating: ***

C42

Start

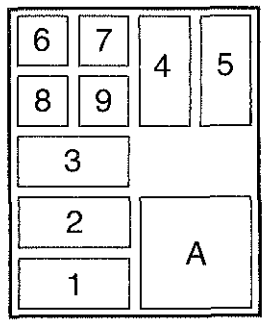


Title: **Century (and a half)**—invented by John H. Conway; published in his book *Winning ways* (Berlekamp *et al.* 1982).

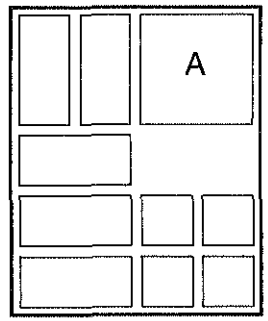
Date: *ca.* 1975; 1982 (*Winning ways*).
Start position: As shown in the diagram—note half-way position of piece 5.
Object: (a) move A to bottom centre
(b) move A to bottom centre and other pieces so that when the whole tray is rotated 180°, it is in the start position (ignoring piece numbers).
Solutions: (a) 100 moves (113)
(b) 150 moves (171).
Ratings: *** (a); **** (b).

C43–C45

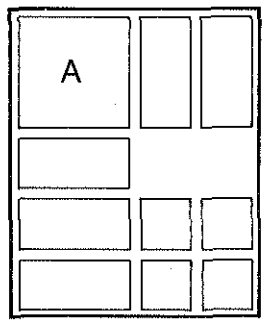
Start



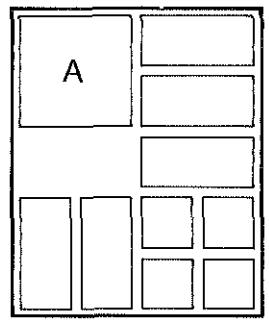
C43 Finish



C44 Finish



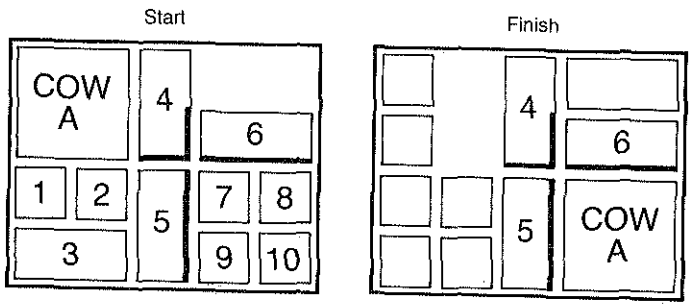
C45 Finish



Title: None.
Date: Patented in the USA (No. 1 017 752) by L. W. Hardy. 1907 (application); 1912 (patented).

Start positions: As shown in C43 for all three puzzles.
Object: Finish as shown in the relevant diagram.
Solutions: C43—55 moves (68)
C44—70 moves (80)
C45—113 moves (127).
Comment: One of the earliest sliding block puzzles having pieces which are rectangular—and a particularly difficult one.
Ratings: *** C43, C44; **** C45.

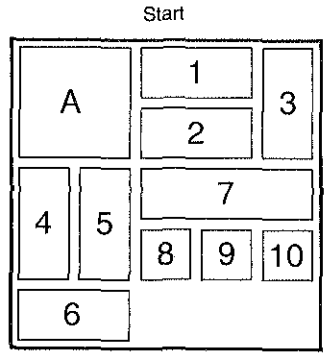
C46



Title: **Fence the Cow**—manufactured by Hanabishi, Japan.
Date: 1984.
Start position: As shown in the diagram.
Object: To get piece A (the cow) down to the bottom right corner and to put the fence pieces 4, 5, 6 around it as shown.
Solution: 71 moves (82).
Rating: **

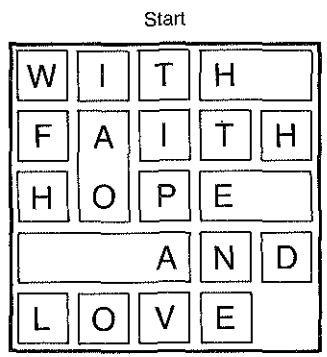
C47

Title: **Marco Polo's Journey**—invented by Dr R. I. Hess, USA.
Date: 1982.
Start position: As shown in the diagram.
Object: Move A (Marco Polo) to the top right corner, bottom



right corner, bottom left corner and back up to top left corner.
Solution: 135 moves (158).
Rating: ***

C48, C49



Title: **Panama Blockade**.
Date: Patented in USA in 1922 (No. 1 412 625) by M. Bulister.
Pieces: In addition to a letter, each of the small squares has the name of a country, and the larger pieces the name of a body of water as follows (reading across left to right from top to bottom): W = England, I = Germany, T = France, H = Atlantic Ocean, F = Russia, A = Gatun Locks, I = Japan, T = Spain,

H = Italy, O = Turkey, P = China, E = Gatun Lake, A = Pedro Miguel Locks, N = Brazil, D = USA, L = Denmark, O = Norway, V = Mexico, E = USA.

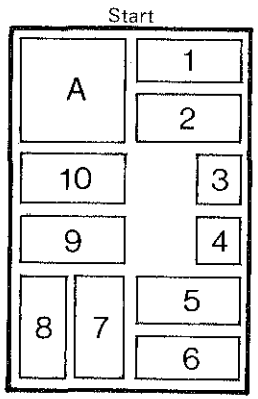
Start positions: As shown in the diagram for all puzzles.

Object: Choose a country, and get it to the bottom right corner, leaving all other pieces in their original positions.

Solutions: Given for two of the more difficult
C48 [Brazil (N)]—68 moves
C49 [England (W)]—80 moves.

Ratings: ****

C50, C51



Title and date: Unknown.

Start position: As shown in the diagram for both puzzles.

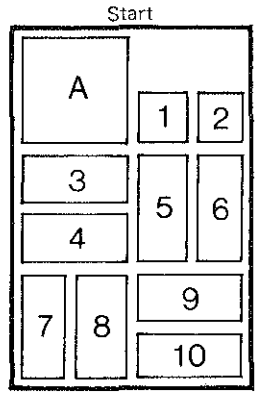
Object: C50—move A to bottom right corner
C51—move A to bottom left corner.

Solutions: C50—41 moves (44)
C51—71 moves (75).

Ratings: **

C52

Title: **11 Block Puzzle**—from *100 puzzles: how to make and how to solve them* by A. Filipiak.



Date: 1942.

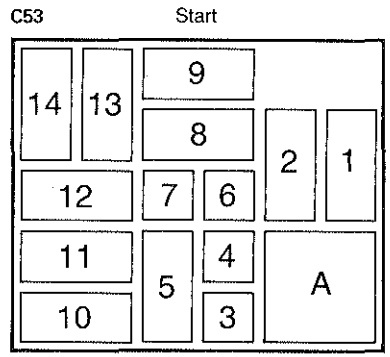
Start position: As shown in the diagram.

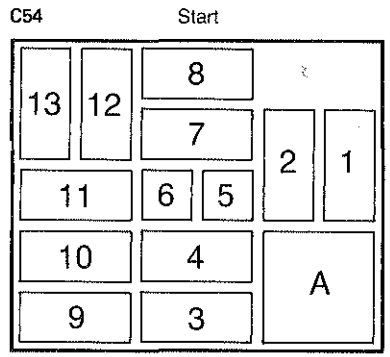
Object: Move A to bottom right corner.

Solution: 90 moves (96).

Rating: **

C53, C54





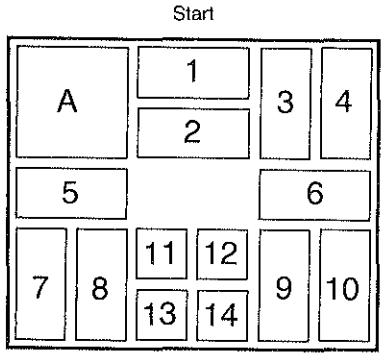
Title: None—manufactured by Hanabishi, Japan.

Date: 1982.

Pieces: Can be turned over to make the second puzzle; in this case the dummy is used and two small square pieces are omitted. The dummy is a rectangular piece that fits into the space.

Start positions: As shown in the diagrams.
Object: Move A to top left corner.
Solutions: C53—55 moves (63)
C54—44 moves (47).
Ratings: *

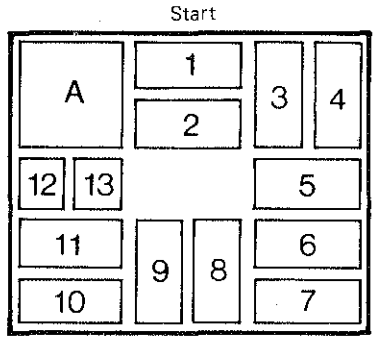
C55



Title: None—one of a series of six designs under the general title of 'Plywood Puzzles' made by Dolphin Games & Toys.
Date: 1940s (?).
Start position: As shown in the diagram.
Object: Move A to bottom right corner.
Solution: 48 moves (55).
Rating: *

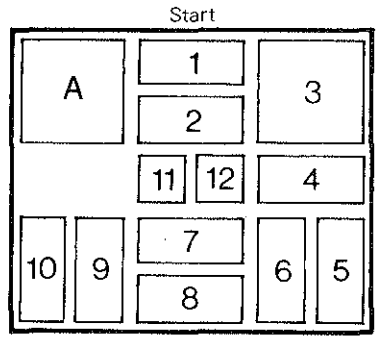
C56, C57

Titles and dates: **Ching Foo** (1926); USA Patent No. 1 663 568 by J. M. Schneider (1928); **Tit-Bits Teaser No. 2** (1928) UK; **The Flying Puzzle**—produced by Standard Trailer Co., USA (1928); **South Pole Expedition** (1934); **The Straight Arrow Puzzle**—by The Oakland Standard Mechanical Company, USA (1981).
Pieces: Flying puzzle pieces were—A = plane; 1 = rain; 2 = hail; 3 = wind; 4 = snow; 5 = clouds; 6 = air; 7 = pocket; 8, 9 = sleet; 10, 11 = water; 12, 13 = fog. Top left corner was marked New York, bottom right corner Paris.



Start positions: As shown in the diagram for both puzzles.
Object: C56—move A (plane) to bottom right corner.
C57—move A to bottom right corner and arrange the rest of the pieces so that when turned upside down, the puzzle is in the start position (ignoring piece numbers).
Solutions: C56—55 moves (59)
C57—129 moves (133).
Ratings: * C56; *** C57.

C58



Titles: **Tit-Bits Teaser No. 5; The Train Puzzle.**
Date: 1931 (UK patent and 'Tit-Bits teaser').

Pieces: Original pieces were—A = Royal Scot; 1 = Carlisle; 2 = Lancaster; 3 = Willesden; 4 = Preston; 5 = Bletchley; 6 = Rugby; 7 = Crewe; 8 = Stafford; 9 = Warrington; 10 = Wigan; 11, 12 = Signal. Top left corner was marked Glasgow, bottom right corner London.

Start position: As shown in the diagram.

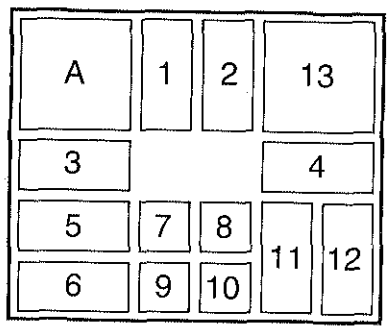
Object: Move A (Royal Scot) to bottom right corner.

Solution: 59 moves (61).

Rating: *

C59

Start



Title: None—one of a series of six designs under the general title of 'Plywood Puzzles' made by Dolphin Games & Toys.

Date: 1940s (?).

Start position: As shown in the diagram.

Object: Move A to bottom right corner.

Solution: 61 moves (67).

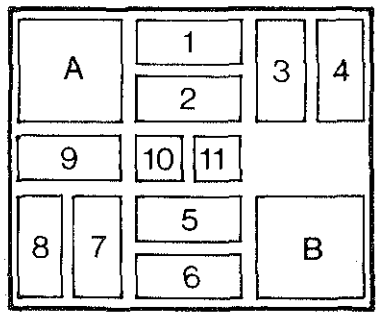
Rating: *

C60, C61

Title: **Century of Progress Puzzle**—USA.

Date: Late 1920s or early 1930s.

Start



Pieces: Original pieces were A = East; B = West; 1 = airplanes; 2 = automobiles; 3 = world; 4 = wonders; 5 = machinery; 6 = inventions; 7 = progress; 8 = century; 9 = Chicago; 10 = you; 11 = see.

Start positions: As shown in the diagram—for both puzzles.

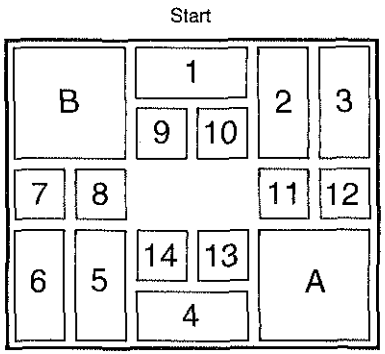
Object: C60—move A (East) to bottom left corner
C61—move B (West) to bottom left corner.

Solutions: C60—37 moves (40)
C61—42 moves (44).

Ratings: *

C62, C63

Start



Title: **Little Hippo**—devised by Dr Kobayashi, Professor at Kagawa University, Japan, and manufactured by Hanayama, Japan.

Date: 1983.

Start positions: As shown in the diagram for both puzzles.

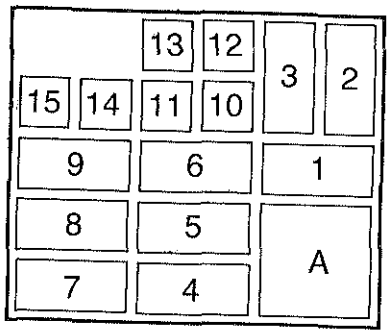
Object: C62—move A to top left corner
C63—move A to top left corner, B to bottom right corner and all the other pieces in their original positions.

Solutions: C62—47 moves (58)
C63—84 moves (102).

Ratings: * C62; ** C63.

C64–C66

Start



Title: **Super Express Bullet Train Game**—designed by Dr Kobayashi, Professor at Kagawa University, Japan and manufactured by Hanayama, Japan.

Date: 1982.

Pieces: Represent various items connected with railways—piece A is the train itself.

Start positions: C64, C65—as shown in the diagram
C66—as in the diagram except that piece 15 should be immediately above piece 14.

Object: C64—move A to top left corner.
C65—move A from Tokyo (bottom right corner) through all the stations (e.g. Ueno) on the right and top edges of the tray, to Morioka (top left corner). In other words piece A must never leave the edges of the puzzle.

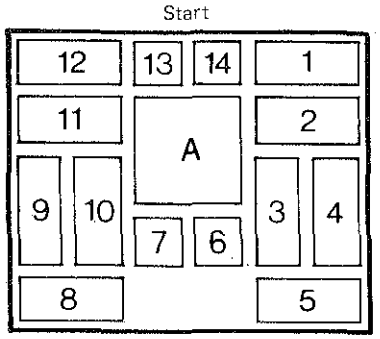
C66—move A from Tokyo (bottom right corner) through all the stations (e.g. Nagoya, Kyoto, Hiroshima) on the bottom and left edges of the tray, to Hakata (top left corner). Piece A must never leave the edges of the puzzle.

Solutions: C64—47 moves (54)
C65—72 moves (85)
C66—78 moves (89).

Ratings: *C64; **C65, C66.

C67

Start



Title: **George Washington's Career** (see Plate VI).

Date: Late 1920s or early 1930s.

Pieces: Original pieces were—A = George Washington; 1 = integrity; 2 = virtue; 3 = liberty; 4 = independence; 5 = brave; 6, 14 = peace; 7, 13 = war, 8 = loyal; 9 = just; 10 = fair; 11 = noble; 12 = honest. Top right corner was marked Trenton, bottom right Valley Forge, bottom left Burgoyne's surrender and top left Presidency.

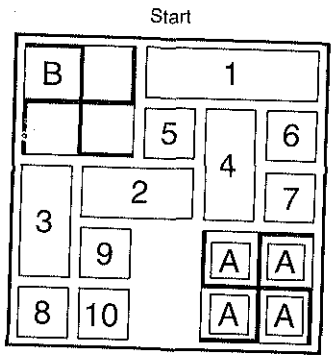
Start position: As shown in the diagram.

Object: Move A (George Washington) to top right corner; from there to bottom right; to bottom left; to top left.

Solution: 122 moves (144).

Rating: **

C68



Title: Unknown—from *Academy of Recreational Mathematics Magazine*, Japan.

Date: 1980.

Pieces: Refer to C69, C70 for a description of the two large squares (pieces A and B) and the four small squares carried by A. The four small squares each have one of the symbols of the four card suits. Note that in C69 and C70 the two large squares are identical, while here they are a mirror image of one another.

Start position: As shown in the diagram.

Object: Transfer the four small squares—spades, hearts, diamonds, and clubs—from piece A to piece B (pieces must be adjacent to do this) and then return piece B to the top left corner.

Solution: 124 moves (144). Sliding the four small squares from piece A to piece B count as moves.

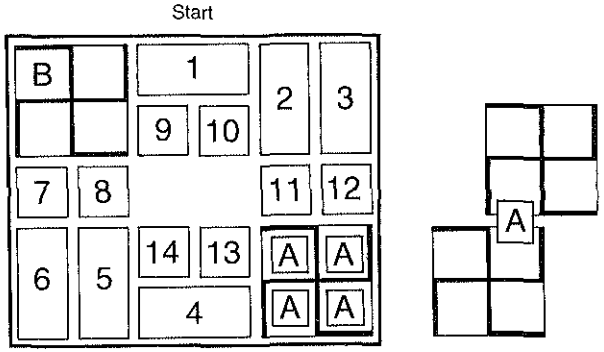
Rating: ***

C69, C70

Title: **Lost Pygmies**—devised by Dr Kobayashi, Professor at Kagawa University, Japan, and manufactured by Hanayama, Japan (see Plate V).

Date: 1983.

Pieces: Exactly the same as for puzzle C62 (Little Hippo) except for the two large squares A and B, which each



have four 'sunken ledges'—see diagram. The raised edges are in the form of a swastika. At the start, piece A carries four small squares, which can be slid across to piece B when they are partly adjacent to each other (see diagram). To slide all four small squares across, it is necessary to bring pieces A and B together four times, once for each face.

Start positions: As shown in the diagram for both puzzles.

Object: C69—transfer all the 'pygmies' (small squares)—Samson, Richard, Carl and Robin—from piece A to piece B and then return piece B to Pygmyland (the top left corner).
C70—as in C69 and return all other pieces to their original positions.

Solutions: C69—103 moves (129)
C70—130 moves (164).
Sliding the four small squares from piece A to B count as moves.

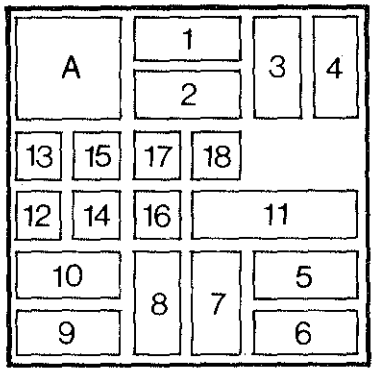
Ratings: ***

C71

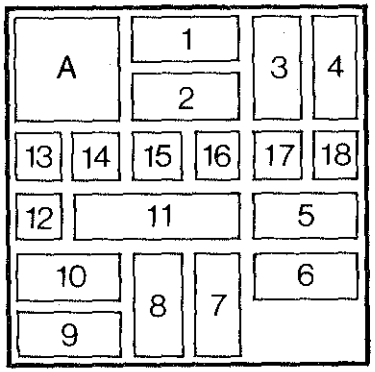
Titles: **Railroadman's Rally; 19 Block Puzzle** (C71a)—from *100 puzzles: how to make and how to solve them* by A. Filipiak; **Pandemonium** (C71b), manufactured by Pentangle.

Dates: Pre-1940 (Railroadman); 1942 (Filipiak); 1970s (Pandemonium).

a Start



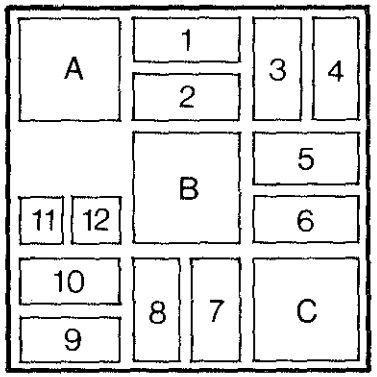
b Start



Start positions: As shown in the diagrams.
Object: Move A to bottom right corner.
Solution: 103 moves (121) for both puzzles.
Ratings: ***

C72, C73

Start



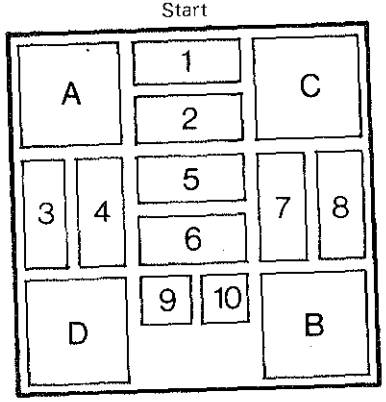
Titles: **Sliding Block Puzzle**—by S. S. Adams (USA); **Tenderfoot's Caution**—UK; **ABC Puzzle; 15 Block Puzzle** from *100 puzzles: how to make and how to solve them* by A. Filipiak; **Countdown**—manufactured by Pentangle, UK.

Dates: 1927 (Sliding Block); late 1920s or 1930s (Tenderfoot); 1942 (Filipiak); 1970s (Countdown).

Start positions: As shown in the diagram for both puzzles.
Object: C72—exchange pieces A and C, (B to remain in the centre).
C73—exchange pieces A and C with all other pieces finishing in their start positions.
Solutions: C72—130 moves (136)
C73—140 moves (148).
Ratings: ***

C74, C75

Start



Titles: **Model Garage** (C74)—from *Popular Science Magazine*, USA; **Spotcheck** (C75)—manufactured by Pentangle, UK; **The Blockade Puzzle**, Canada.

Date: October 1932 (Model Garage); 1970s (Spotcheck); 1981 (Blockade).

Pieces: Original pieces were—A = bus; B, C, D = trucks; 1, 2, 3, 4, 5, 6, 7, 8 = cars.

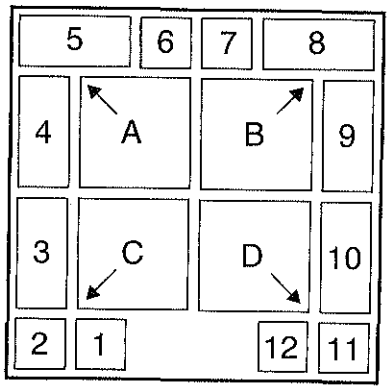
Start positions: As shown in the diagram for both puzzles.
Object: C74—move A to bottom centre.
C75(a)—exchange pieces A with B, C with D.
C75(b)—exchange pieces A with B, and C with D, with all other pieces finishing in their start positions.
Solutions: C74—51 moves (54)

C75(a)—211 moves (222)
C75(b)—213 moves (224).

Ratings: * C74 and *** C75.

C76

Start



Title: **Crossover**—made by Pentangle.
Date: 1984.
Start position: As shown in the diagram.,
Object: To exchange pieces A with D and B with C so that all the arrows point inwards towards each other. The square block of pieces A, B, C, D must be:
(a) situated anywhere;
(b) in the centre;
(c) in the centre and with all the other pieces in their original positions.
Solutions: (a)—154 moves (180)
(b)—181 moves (213)
(c)—209 moves (243).
Rating: ***

6

SLIDING BLOCK PUZZLES WITH NON-CONVEX PIECES, D1–D50

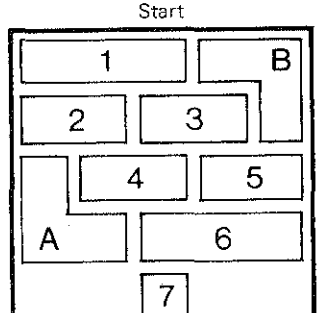
This category of sliding block puzzles contains L-shaped and/or T-shaped pieces. For the majority of the puzzles this increases the difficulty considerably. The L-shaped piece adds an entirely new dimension and is a most restrictive element. The T-shaped piece is even worse. The hardest puzzles in the book are in this chapter and the later ones (D44–D50) present the greatest difficulty and are for experts only. D49 is the ultimate challenge and is one of only two puzzles in the book to warrant a ****** rating. The other is E35.

Try D1, D2, D3, D8 first; then D12–D21; D22–D50 all make excellent puzles for the enthusiast.

For an explanation and discussion on what constitutes a move, refer to the 'moves' section in Chapter 1 (see page 5).

D1–D3

Start



Titles: **Ma's Puzzle**—manufactured by Standard Trailer Co., USA (see Plate VII); **Spirit of '76**—USA; also **Wooden Puzzle**—made by Hanabishi, Japan.

Dates: Patented in USA in 1927 (No. 1 633 397) by C. L. Diamond; 1983 (Wooden Puzzle).

Pieces: Original pieces were—A = My Boy; B = Ma; 1 = No Work; 2 = Danger; 3 = Broke; 4 = Worry; 5 = Trouble; 6 = Homesick; 7 = Ill.

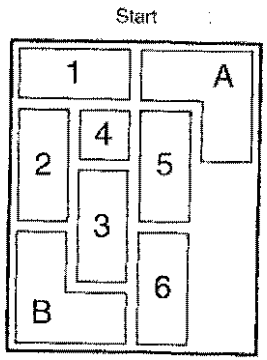
Start positions: As shown in the diagram for all three puzzles.

Object: D1—move A to join B to form rectangle in top right corner (original).
D2—move A to join B to form rectangle anywhere.
D3—move A to join B to form horizontal rectangle in top right corner.

Solutions: D1—23 moves (26)
D2—19 moves (22)
D3—37 moves (41).

Ratings: *

D4



Title: **Mini Ma's Puzzle**—adapted from an idea by Dr R. I. Hess, USA.

Date: *ca.* 1980.

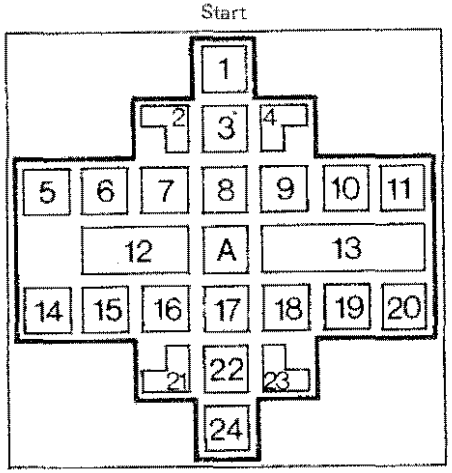
Start position: As shown in the diagram.

Object: Move A to join B as a rectangle anywhere.

Solution: 28 moves (33).

Rating: **

D5



Title: None.

Date: Patented in USA in 1904 (No. 771 514) by W. H. Wehner.

Pieces: Original pieces were shown in the patent as two teams of American Football positions. A was the ball and the two rectangular pieces were the umpire and the referee.

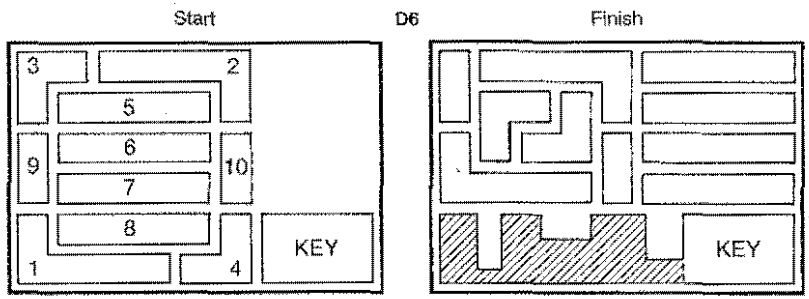
Start position: As shown in the diagram.

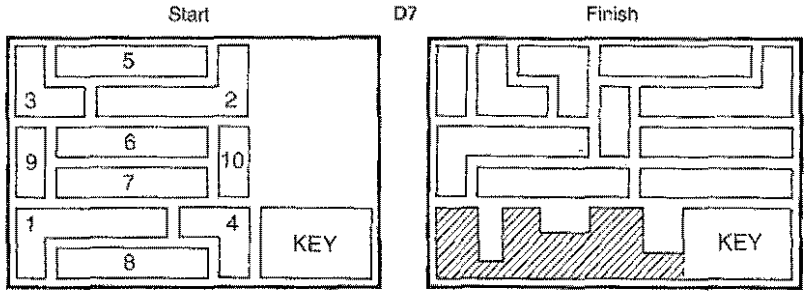
Object: Move A to top or bottom centre.

Solution: 36 moves (37).

Rating: *

D6, D7





Title: **Key Puzzle**—invented by Kunio Kumoi (Japan); manufactured by Hanabishi, Japan.

Date: 1983.

Pieces: This is an unusual puzzle in that there is a 'key' piece which can only be removed when all the pieces have been moved to the top two thirds of the tray. The shaded part of the key is set into the bottom of the tray and the other pieces are free to slide on top of this part of the key. The two large 'L' shaped pieces need to be turned over for D7.

Start positions: As shown in the diagrams.

Object: To remove the key, which can only be done by sliding all the other pieces into the top two thirds of the tray—see diagrams. No turning (rotating) of the pieces is allowed.

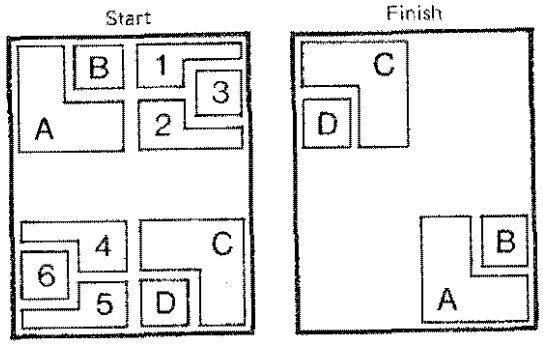
Solutions: D6—25 moves (31). Removing key does not count as a move.
D7—20 moves (31). Removing key does not count as a move.

Ratings: **

D8

Title: None—submitted to *Games Digest Magazine*, USA, by E. B. Escott.

Date: August 1938.



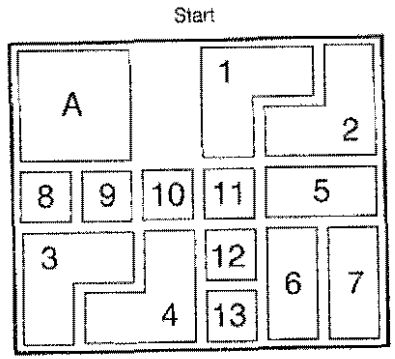
Start position: As shown in the diagram.

Object: Finish as shown.

Solution: 48 moves (61).

Rating: ***

D9



Title: None—one of a series of six designs under the general title of 'Plywood Puzzles' made by Dolphin Games & Toys.

Date: 1940's (?).

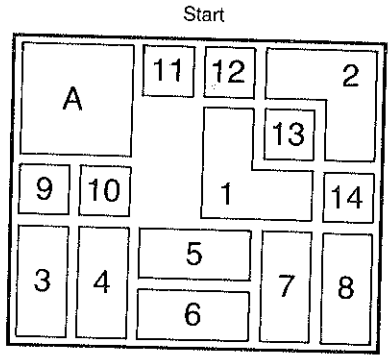
Start position: As shown in the diagram.

Object: Move A to bottom right corner.

Solution: 63 moves (73).

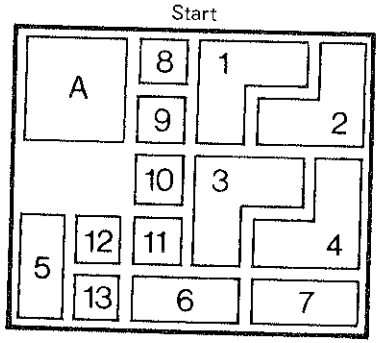
Rating: **

D10



Title: None—one of a series of six designs under the general title of 'Plywood Puzzles' made by Dolphin Games & Toys.
Date: 1940s (?).
Start position: As shown in the diagram.
Object: Move A to bottom right corner.
Solution: 76 moves (87).
Rating: ***

D11

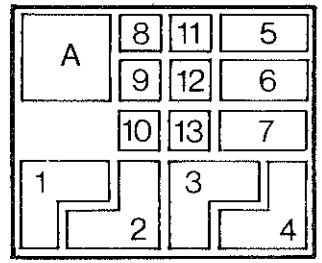


Title: **Dingbat**—manufactured by Dr Hex Association, USA.
Date: 1930s.
Pieces: Original pieces were—A = Dingbat. The top left corner was marked Detroit and bottom right was Washington.

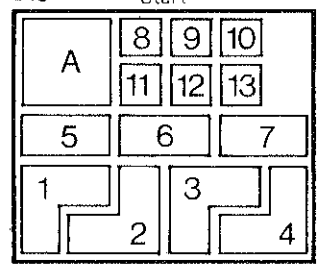
Start position: As shown in the diagram.
Object: Move A to bottom right corner.
Solution: 98 moves (114).
Rating: ***

D12–D23

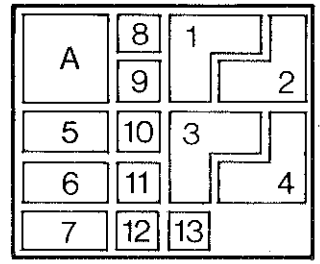
D12



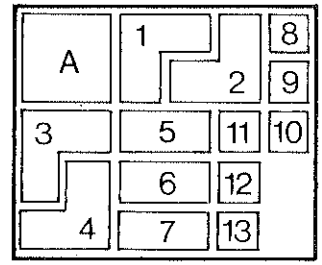
D13



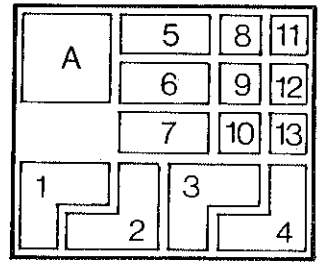
D14



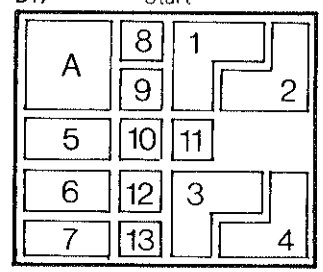
D15

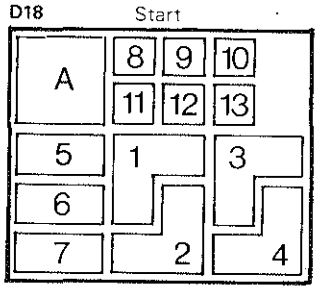


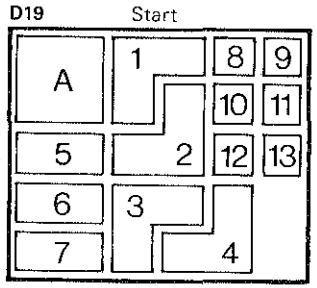
D16

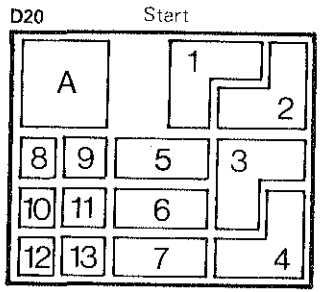


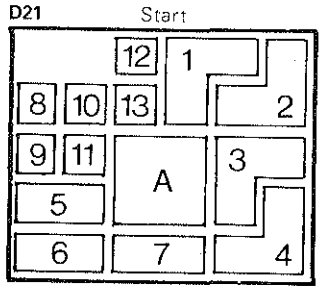
D17

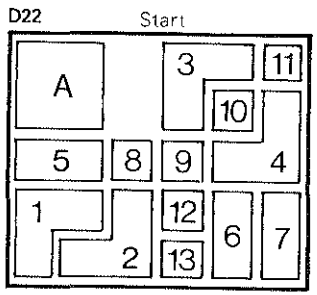


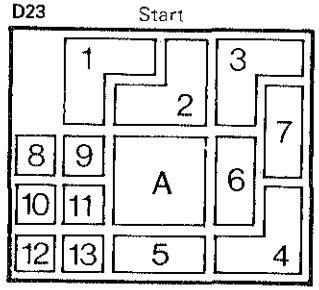












Titles: **The Traffic Jam Puzzle** (see Plate VII); **Tit-Bits Teaser No. 4** (D22).

Dates: Patented in USA in 1928 (No. 1 683 014) by F. L. Babcock; patented in UK in 1930 (Tit-Bits teaser No. 4).

Pieces: Original pieces were—(Traffic jam) A = My Car; 1 = Moving Van; 2 = Ice Truck; 3 = Street Car; 4 = Auto Bus; 5 = Chevrolet; 6 = Hudson; 7 = Oakland; 8 = Buick; 9 = Dodge; 10 = Ford; 11 = Nash; 12 = Paige; 13 = Reo. (Tit-Bits) A = My Car; 1 = Van; 2 = Lorry; 3 = Tramcar; 4 = Bus; 5 = Humber; 6 = Bentley; 7 = Singer; 8 = Bean; 9 = Riley; 10 = Swift; 11 = Morris; 12 = Austin; 13 = Rover. In both puzzles the top left corner was marked Office and the bottom right Home.

Start positions: As shown in the diagrams.

Object: Move A (my car) to bottom right corner (home) for all 12 puzzles.

Solutions:

D12—43 moves (51)	D18—75 moves (89)
D13—52 moves (61)	D19—99 moves (115).
D14—58 moves (67)	D20—80 moves (94)
D15—60 moves (69)	D21—128 moves (150)
D16—57 moves (64)	D22—132 moves (147)
D17—73 moves (85)	D23—223 moves (257)

Ratings: **D12–D18; ***D19–D21; ****D22; *****D23.

D24–D43

Title: **Block-10**—designed by Minoru Abe, Japan.

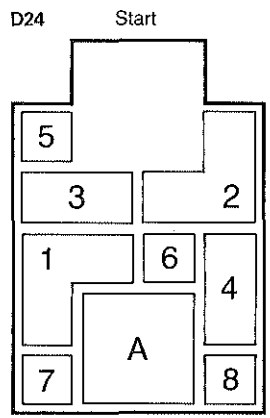
Date: 1985.

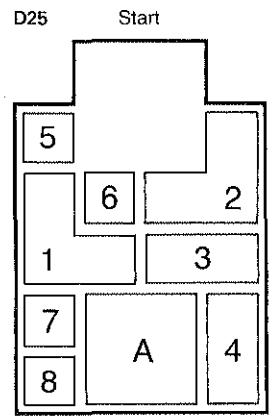
Start positions: As shown in the diagrams.

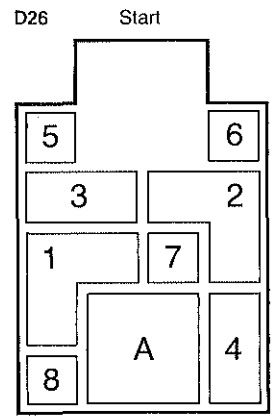
Object: Move A to top centre.

Solutions:

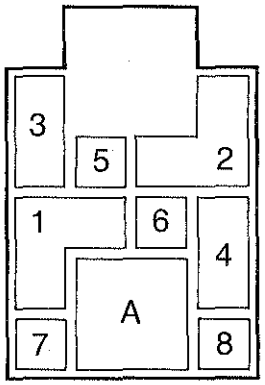
D24—26 moves (35)	D27—51 moves (60)
D25—37 moves (58)	D28—27 moves (38)
D26—34 moves (49)	D29—26 moves (37)



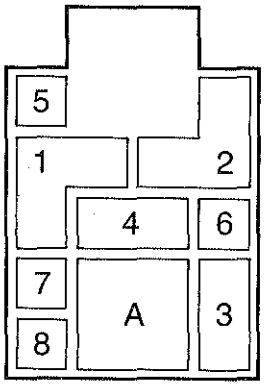




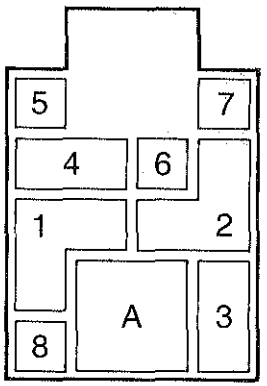
D27 Start



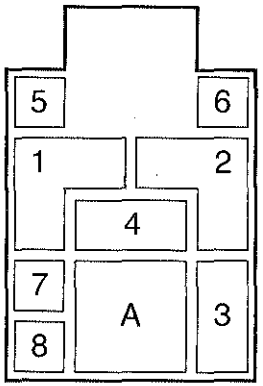
D28 Start



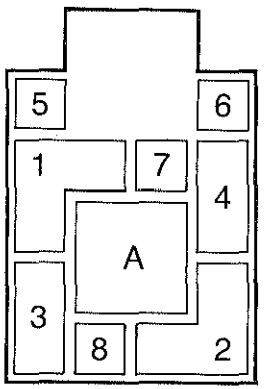
D29 Start



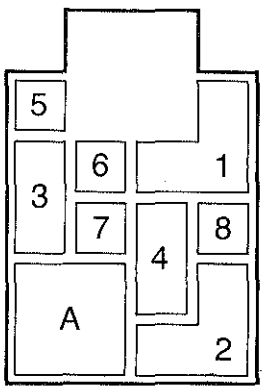
D30 Start



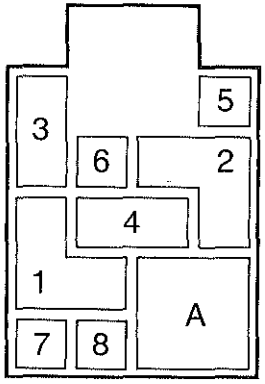
D31 Start



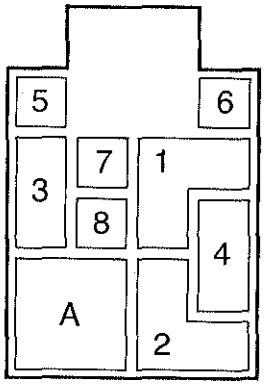
D32 Start



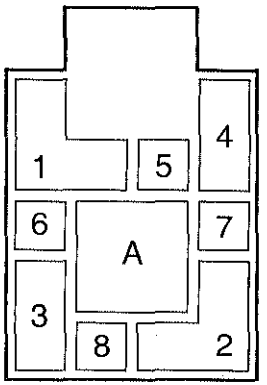
D33 Start



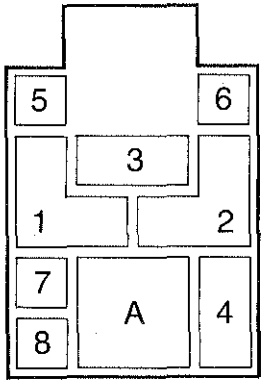
D34 Start



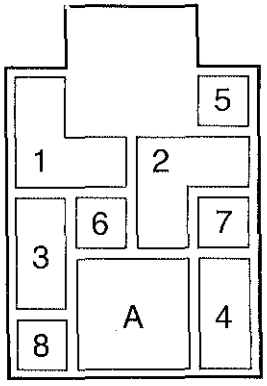
D35 Start



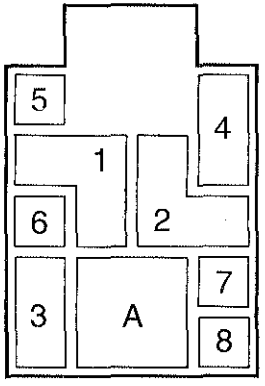
D36 Start



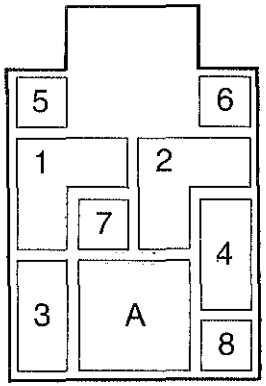
D37 Start



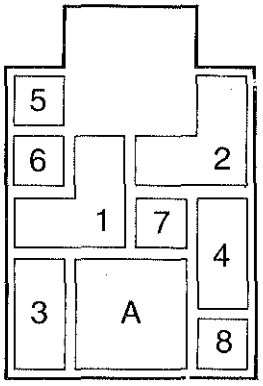
D38 Start



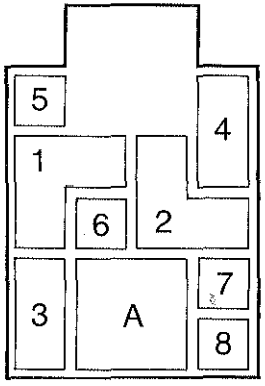
D39 Start



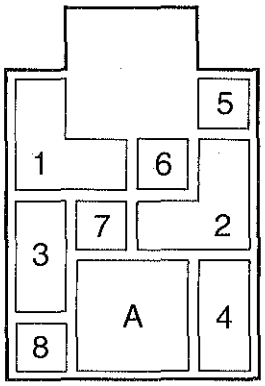
D40 Start



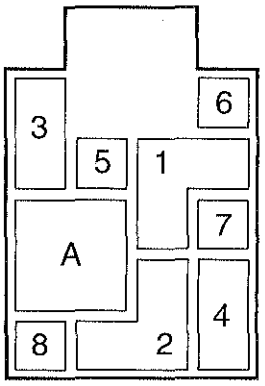
D41 Start



D42 Start



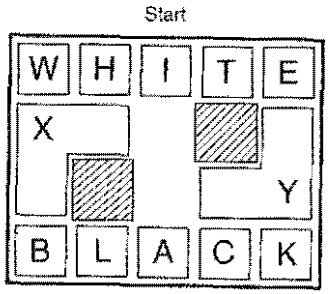
D43 Start



D30—34 moves (47)
D31—36 moves (47)
D32—38 moves (57)
D33—33 moves (48)
D34—36 moves (52)
D35—49 moves (67)
D36—36 moves (49)
D37—52 moves (70)
D38—59 moves (74)
D39—41 moves (61)
D40—42 moves (55)
D41—41 moves (56)
D42—42 moves (59)
D43—31 moves (41).

Ratings: *** (all).

D44



Title: **Neo Black & White**—designed by Minoru Abe, Japan.

Date: 1985.

Start position: As shown in the diagram.

Object: Exchange the positions of WHITE and BLACK so that the top row reads BLACK and the bottom row WHITE. Note the two immovable blocks (shaded areas) which are fixed to the base.

Solution: 198 moves (237).

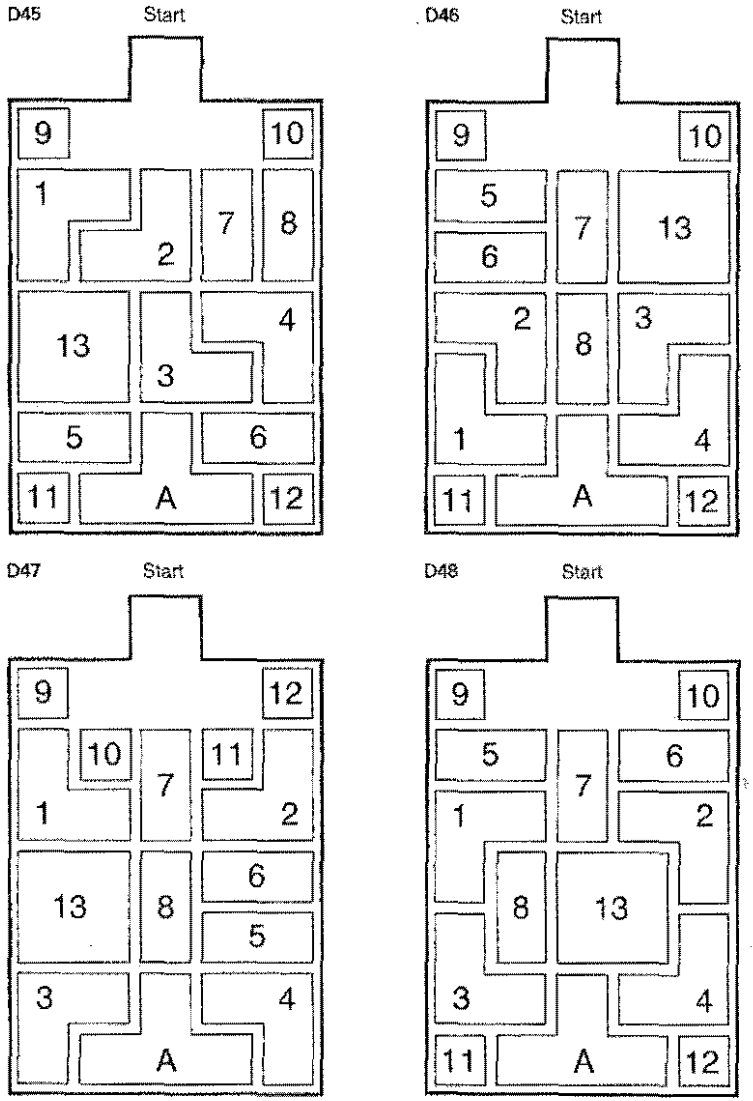
Rating: ****

D45–D48

Title: **Climb Game 15**—designed by Minoru Abe, Japan (see Plate VII).

Date: 1985.

Start positions: As shown in the diagrams.



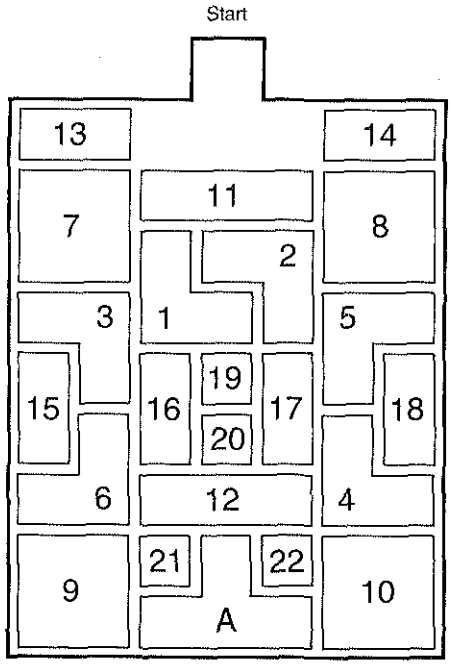
Object: Move A to top centre.

Solutions: D45—133 moves (172)
D46—163 moves (207)
D47—190 moves (246)
D48—203 moves (277).

Comment: These puzzles are exceptionally difficult.

Ratings: ***** (all).

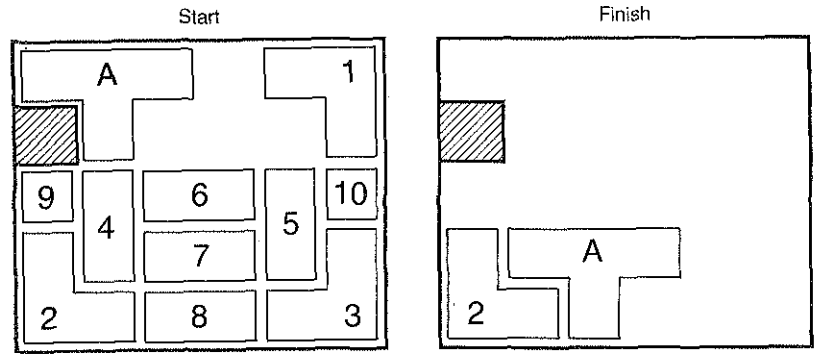
D49



Title: **Climb Pro 24**—designed by Minoru Abe, Japan.
Date: 1985.
Start position: As shown in the diagram.
Object: Move A to top centre.
Solution: 474 moves (568).
Comment: This puzzle is exceptionally difficult and probably represents the ultimate in sliding block challenges.
Rating: ******

D50

Title: **The Sliding T puzzle**—invented by Dr R. I. Hess, USA.
Date: 1982.
Start position: As shown in the diagram.



Object: Move A to the bottom of the puzzle next to piece 2 as shown in the diagram. Note the immovable block (shaded).
Solution: 159 moves (186).
Rating: *****

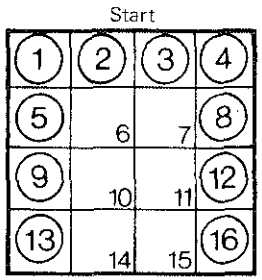
7

RESTRICTED ROUTE SLIDING BLOCK PUZZLES, E1–E35

In this group, either the pieces or the puzzle itself have some restriction as to movement. The group includes: puzzles having criss-cross/interweaving routes; pieces that may only travel in one direction; pieces that can only move lengthwise, not sideways; pieces that cannot move at all; pieces that cannot enter all parts of the puzzle; pieces that cannot touch other pieces of the same colour; pieces that cannot check (be on the same line as) other pieces of the same colour; and pieces that cannot be placed above a piece having a lower number. There is even a chess sliding block puzzle, in which white must give checkmate in 165 moves! Some of the puzzles in this category must be solved in the given number of moves and, as in Chapter 4, they have an additional * rating in brackets for this purpose.

The most interesting puzzles are: E1, E2, E5, E6, E7, E14, E16, E17, E18, E20, E22, E24, E28, E29, E30, E32, E33.

E1



Title: **The Ten Prisoners**—from *Amusements in mathematics* by H. E. Dudeney.

Date: 1917 or earlier.

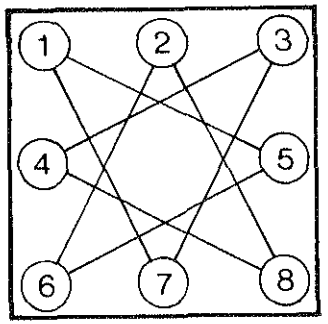
Start position: As shown in the diagram.

Object: Move four counters (prisoners) diagonally or orthogonally, so as to give 16 rows (orthogonally and diagonally) with an even number of counters in each.

Solution: 4 moves. Note that pieces can move any distance.

Rating: **

E2



Titles: **The Four Frogs**—from *Amusements in mathematics* by H. E. Dudeney. Also **The Mercury Puzzle**—one of a series under the general title of 'Magnetic Spaceman'.

Dates: 1917 or earlier. However, Dudeney took this puzzle from Guarini (1512) who used four knights on a chess board. 1984 (Mercury).

Start position: Two white counters on 1 and 3, two black on 6 and 8.

Object: Exchange the positions of the white and black counters in seven moves, moving only along straight lines and stopping only on circles. A counter may move any distance per move, i.e. to two or more circles, provided they are not occupied by another counter.

Solution: 7 moves.

Comment: The real problem is not so much to find a solution, as to find the seven-move solution.

Rating: **

E3

Title: **Green Witch**—made by Four Generations, USA. One of a series of 'Penny Puzzlers'.

Date: 1974.

Start position: The same as for the previous puzzle (E2) except that three white counters are placed on 1, 2, 3 and three black on 6, 7, 8.

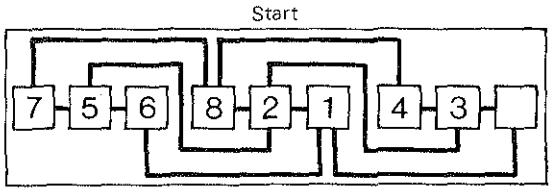
Object: Exchange the positions of the white and black counters, moving only along straight lines and stopping only on circles. A counter may move any distance per move i.e. to two or more circles, provided they are not occupied by another counter.

Solution: 7 moves.

Comment: The real problem is not so much to find a solution as to find a seven-move solution.

Rating: *

E4



Titles: **The Man of Law's Puzzle**—submitted to *The London Magazine* by H. E. Dudeney, and later published in his book *The Canterbury puzzles*. Also **The Warden's Problem.**

Date: 1902.

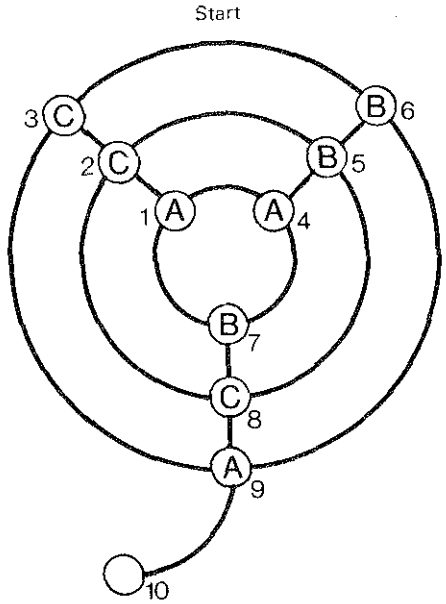
Start position: Place counters 1–8 on the squares as shown.

Object: Arrange the pieces in correct numerical sequence 1–8 with the blank space on the right. Move only along the lines from square to square.

Solution: 26 moves.

Rating: ** (*** in minimum moves).

E5



Titles: **A Railway Puzzle**—from *Amusements in mathematics* by H. E. Dudeney. Also **The Saturn Puzzle**—one of a series under the general title of'Magnetic Spaceman'.

Dates: 1917 or earlier (Dudeney); 1984 (Magnetic Spaceman).

Start position: Place three counters of one colour on A; three of a second colour on B; three of a third colour on C.

Object: Arrange the pieces so as to have three different colours on each circle and each straight line. Move only along lines from circle to circle.

Solution: 9 moves.

Rating: * (** in minimum moves).

E6

Title: **Orbit**—manufactured by Strataboard Ltd., (see Plate VIII).

Date: 1976.

Start position: Using the same diagram as for the previous puzzle

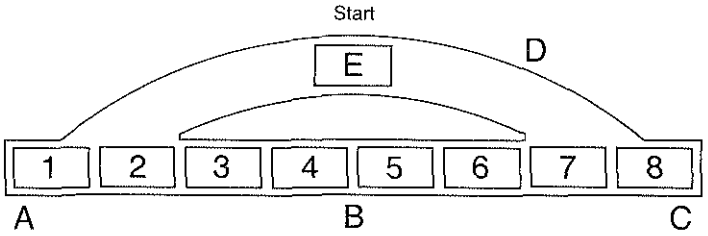
(E5), place three counters of one colour on the outer circle, three counters of a second colour on the middle circle, and three counters of a third colour on the inner circle.

Object: Arrange the pieces so as to have three different colours on each circle and each straight line. Move only along lines from circle to circle.

Solution: 16 moves.

Rating: * (*** in minimum moves).

E7



Titles: **Chifu-Chemulpo or Russo-Jap Railway Puzzle**—manufactured by S. G. & Co., UK (see Plate VIII). Also: **Switchback Puzzle**.

Dates: *ca.* 1905. Patented in 1903 (US Patent No. 743 015) by E. S. Mowry. (Registered Design No. 430 064).

Pieces: One engine (E) and eight cars (1–8), which are all the same length. Main line sections A and C can each take a car, while allowing another to switch onto the branch line D. Main line B and branch line D can each take four cars whilst allowing others on the switches.

Start position: As shown in the diagram.

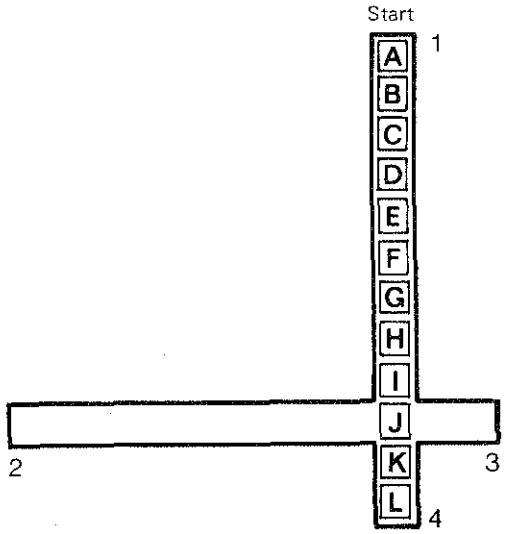
Object: Reverse the positions of all the cars on the main line B (so that car 1 is on the right and 8 on the left) leaving the engine on the branch line D. Note that a move is only counted when a car or the engine changes from the branch line to the main line or vice versa. No other moves count. The instructions invite a solution in 20 moves.

Solution: 26 moves.

Comment: Although this appears to be a railway shunting puzzle, it falls, strictly speaking, within the definition of a sliding block puzzle. This is because every car (piece) can freely move by itself. In railway shunting puzzles only the engine(s) can cause the cars (other pieces) to move.

Rating: * (** in minimum moves).

E8, E9



Titles: **A Chinese Switch Word Puzzle** (E9)—from *Cyclopedia of 5000 puzzles, tricks and conundrums* by S. Loyd.

Date: 1914 or earlier.

Object: E8—find a word which can be transposed from vertical to horizontal arrangement in minimum moves.

Solution: 12 moves.

Rating: **

Alternative puzzle: E9.

Start position: As shown in the diagram.

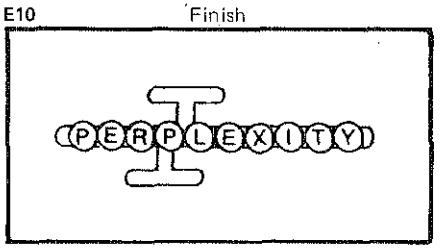
Object: Arrange the letters in alphabetical order along the horizontal slot with A at left and L at right. Note that pieces can move any distance.

Solution: 40 moves.

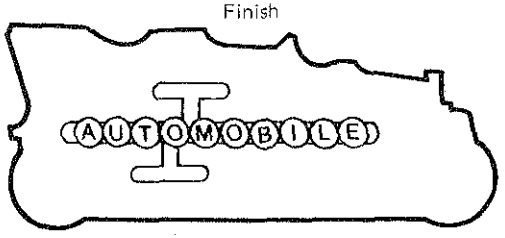
Rating: *

E10, E11

E10



E11



Titles: **Perplexity Puzzle** (E10); **The Wonderful Automobile puzzle** (E11) (see Plate VIII).

Dates: Patented in 1900 by R. M. Schaffer USA (No. 642 374) The letters used in the patent were COPENHAGEN. A different version was produced for the Louisianna Purchase Exposition in 1904. Perplexity was listed in a 1919 catalogue.

Pieces: Consist of 'buttons' sliding in a slot—they cannot be moved from the puzzle.

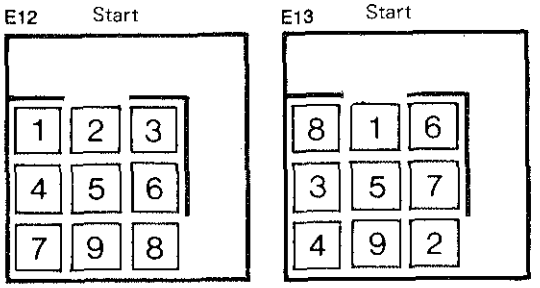
Start positions: with the word PERPLEXITY or AUTOMOBILE spelt backwards.

Object: To reverse the letters to spell the correct word. The

problem would appear to be easy until one tries to move the last letter on the right out of the middle slot.

Ratings: **

E12, E13



Title: **Square Around**—invented by Trevor Truran, published in *Games and Puzzles Magazine*.

Date: Winter 1978.

Start positions: As shown in the diagrams.

Object: Get all the pieces in order (as Start 1, but with 8 and 9 reversed). Note the immovable partition which forms a corridor. Only one piece may be in the corridor at any one time and it cannot re-enter the puzzle through the same door as it left, but must go through the other door. Note that pieces can move any distance.

Solutions: E12—21 moves
E13—38 moves.

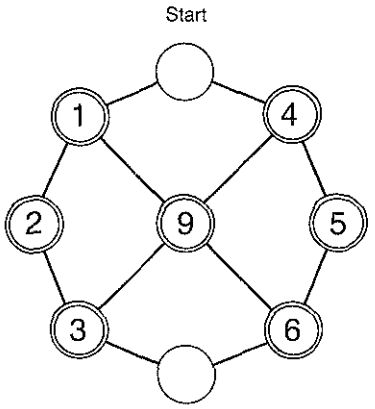
Ratings: ** (*** in minimum moves).

E14

Titles: **The Klondyke Miners' Puzzle** (see Plate VII); **Fliegle Flugel Puzzle** manufactured by Pentangle.

Dates: *ca.* 1905 (Klondyke)—UK Patent No. 1199; 1970s (Fliegle).

Pieces: Three British miners (1, 2, 3), three American miners (4, 5, 6) and an Agent (9).



Start position: As shown in the diagram.

Object: Moving along the lines only, exchange the positions of the three British miners (1, 2, 3) and the American miners (4, 5, 6). Miners can only move forwards, i.e. in the direction of their destination, and only along the lines to a vacant claim (circle). The first two moves must be made by miners (1, 2, 3, 4, 5, 6) and then moves must alternate agent (9), miner, agent, miner etc.

Solution: 30 moves.

Rating: **

E15

Title: **Jeepers!**—from *The real puzzle book* by D. Rubin.

Date: 1984.

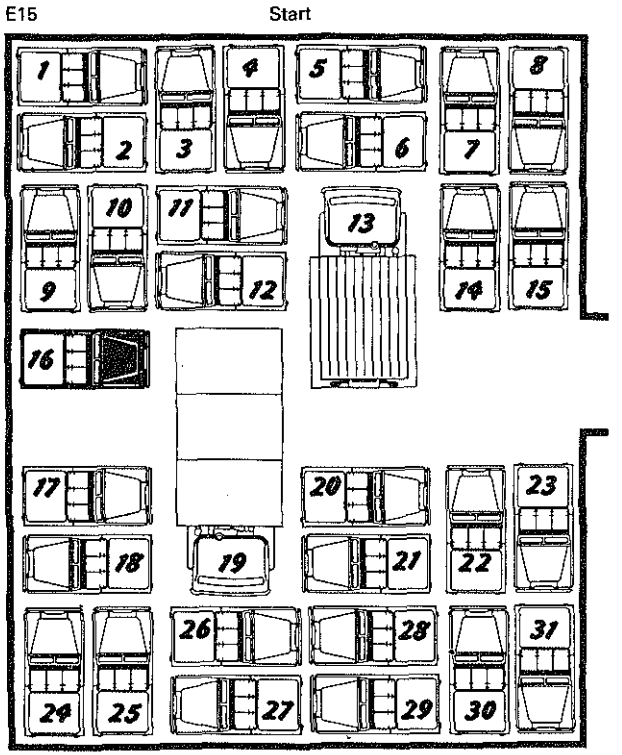
Pieces: Represent jeeps and trucks in a garage.

Start position: As shown in the diagram.

Object: Move piece 16 to extreme right (out of garage). Pieces may only move forwards or backwards (as vehicles would), not sideways. Furthermore, they must not turn.

Solution: 25 moves.

Rating: **



E16

Title: **Lots of luck**—from *What's the big idea* (published as *The puzzle book* in UK) by D. Rubin.

Date: 1979.

Pieces: Represent vehicles in a garage.

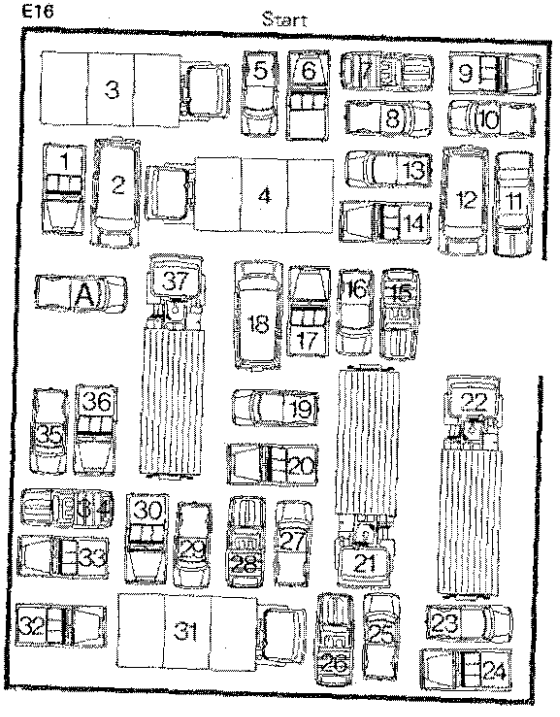
Start position: As shown in the diagram.

Object: Move A to extreme right (out of garage). Pieces may only move forwards or backwards (as vehicles would), not sideways. Furthermore, they must not turn.

Solution: 41 moves.

Rating: **

E16



E17

Title: **The Ambassador Puzzle: Thanks—A Lot**—invented by D. Rubin, published in *TWA Ambassador Magazine*, USA.

Date: May 1980.

Pieces: Represent vehicles in a garage.

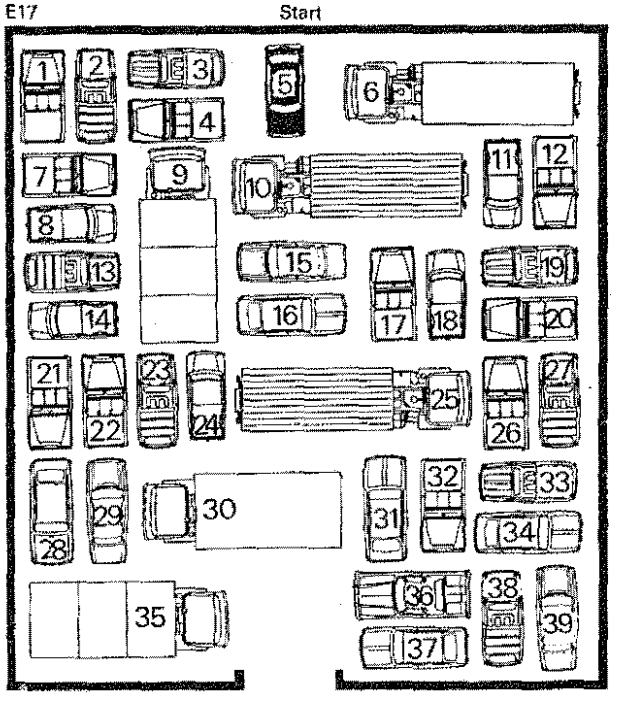
Start position: As shown in the diagram.

Object: Move piece 5 down to the bottom (out of the garage). Pieces may only move forwards or backwards (as vehicles would), not sideways. Furthermore they must not turn.

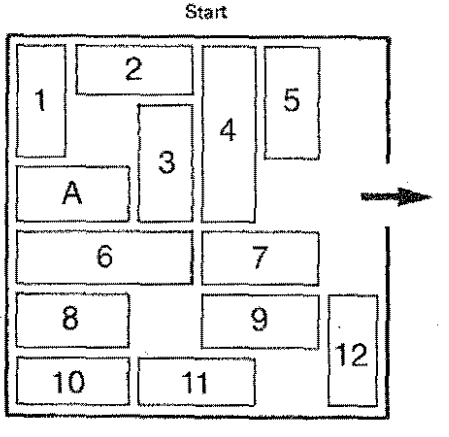
Solution: 59 moves.

Rating: **

E17



E18

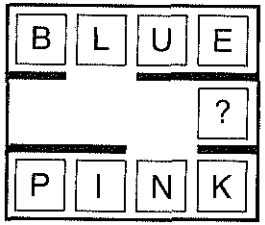


Title: None—invented by N. Yoshigahara and modified by K.Kitajima, Japan; published in *Quark Magazine*, Japan.

Date: May 1985.
Pieces: Represent cars and buses in a garage.
Start position: As shown in the illustration.
Object: Move A to the extreme right (out of the garage). Pieces may only move forwards or backwards (as vehicles would), not sideways. Furthermore they must not turn.
Solution: 23 moves.
Rating: **

E19

Start

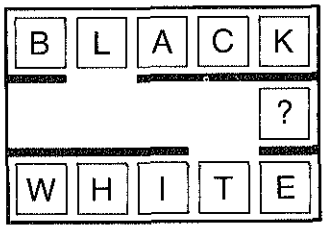


Title: **Pink and Blue**—invented by Nob Yoshigahara, Japan.
Date: 1982.
Pieces: All are the same size except the piece with the question mark, which is a little larger and will not go through the gates into either the top or bottom rows.
Start position: As shown in the diagram.
Object: Put PINK into the bottom row and BLUE into the top row—both in their correct order. Note the immovable barriers between the rows.
Solution: 44 moves.
Rating: * (** in minimum moves).

E20

Titles: **Black and White**—invented by Nob Yoshigahara, Japan; **Comic Scramble**—manufactured by Hanayama, Japan. In the 'Comic Scramble' puzzle the tray is held lengthwise rather than across (as in Black and White)

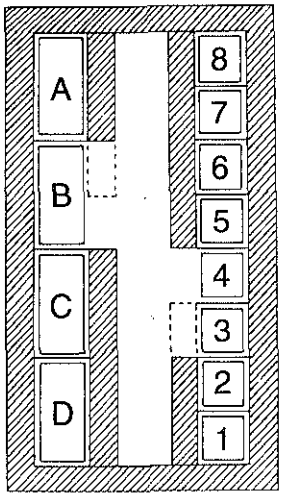
Start



and the pieces represent a man (instead of WHITE) and a woman (instead of BLACK) (see Plate VII).
Date: 1982.
Pieces: All are the same size except for the piece with the question mark, which is a little larger and will not go through the gates into either the top or bottom rows.
Start position: As shown in the diagram.
Object: Put WHITE into the top row and BLACK into the bottom row both in their correct order. Note the immovable barriers between the rows.
Solution: 57 moves.
Rating: * (*** in minimum moves).

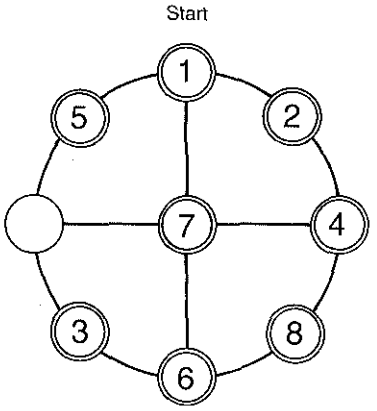
E21

Start



Title: **Container Confusion**—from *Top Puzzles Magazine*.
Date: July/August 1981.
Pieces: Represent containers that have been wrongly stacked in a warehouse.
Start position: As shown in the diagram.
Object: Exchange A–D with 1–8 so that the right side goes from D down to A, and the left side goes from 1 down to 8 starting from the top. The dotted lines represent automatic gates which automatically open for pieces A–D but not for the numbered pieces, which must not cross them.
Solution: 33 moves. Pieces can move any distance per move.
Rating: * (** in minimum moves).

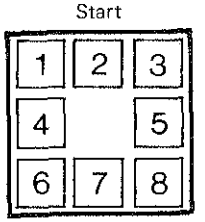
E22



Title: **The Eight Engines**—from *Amusements in mathematics* by H. E. Dudeney.
Date: 1917 or earlier.
Start position: As shown in the diagram.
Object: Move the pieces one at a time, from circle to circle, so as to put them in numerical order round the outside with the centre circle vacant. However, one of the engines (pieces) has broken down and is therefore unable to move. Find out which engine is unable to move.

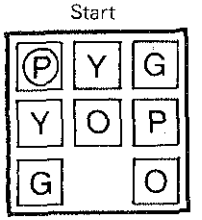
Solution: 17 moves.
Rating: ** (*** in minimum moves).

E23



Title: **Eight Jolly Gaol Birds**—from *Amusements in mathematics* by H. E. Dudeney.
Date: 1917 or earlier.
Start position: As shown in the diagram.
Object: Make a magic square (all orthogonals and the two main diagonals should add up to the same total). One piece (of your choice) must not move.
Solution: 19 moves.
Rating: ** (*** in minimum moves).

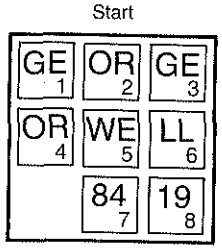
E24



Title: **Mystic**—manufactured by Richard Appel Inc., USA.
Date: 1943.
Pieces: P = purple; Y = yellow; G = green; O = orange.
Start position: As shown in the diagram.
Object: Move (P) (top left corner) to bottom right corner. Pieces of the same colour may never touch each other orthogonally.

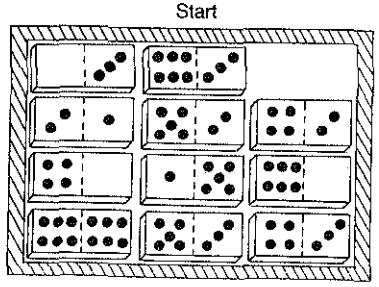
Solution: 22 moves.
Rating: * (** in minimum moves).

E25



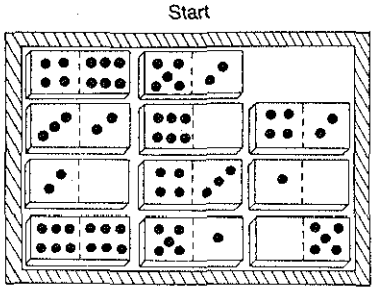
Title: **George Orwell Puzzle**—invented by Trevor Truran, published in *Computer Talk Magazine*.
Date: 1984.
Start position: As shown in the diagram.
Object: Exchange the positions of 84 and 19 so that the puzzle reads: George Orwell 1984. A letter on one piece must *not* be moved adjacent to the same letter on another piece. For example, piece 1 (GE) and piece 5 (WE) can not go above one another as the Es would then be adjacent. On the other hand, they could go alongside each other as there would be a G or a W between the Es.
Solution: 32 moves.
Rating: * (** in minimum moves).

E26



Title: **Domino Slide 1**—from *Top Puzzles Magazine*.
Date: June/July 1981.
Pieces: Eleven dominoes as shown in the diagram.
Start position: As shown in the diagram.
Object: Move the double six to the top right corner and leave the space in the bottom left corner. At the end of each move, neither number on the domino moved must be directly adjacent to the same number on a touching domino. Similar diagonal numbers are permitted.
Solution: 41 moves.
Rating: **

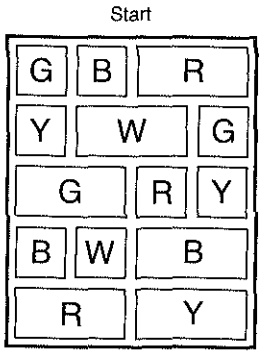
E27



Title: **Domino Slide 2**—from *Top Puzzles Magazine*.
Date: October/November 1981.
Pieces: Eleven dominoes as shown in the diagram.
Start position: As shown in the diagram.
Object: Move the double six to the top right corner and leave the space in the bottom left corner. At the end of each move, neither number on the domino moved must be directly adjacent to the same number on a touching domino. Similar diagonal numbers are permitted.
Solution: 35 moves.
Rating: **

E28

Title: None—invented by Trevor Truran, published in *Games and Puzzles Magazine*.



Date: Summer 1981.

Pieces: The pieces are all coloured: G = green; B = blue; R = red; Y = yellow; W = white.

Start position: As shown in the diagram.

Object: Remove the red piece in the top right corner and get the red piece in the bottom left corner up to the top right corner; continue moving pieces until there is a space in the bottom left corner into which the removed red piece can be placed. At no time can any piece slide past or come to rest against another piece of the same colour—they may, however, touch at their corners.

Solution: 40 moves (46) excluding removing and replacing the red piece.

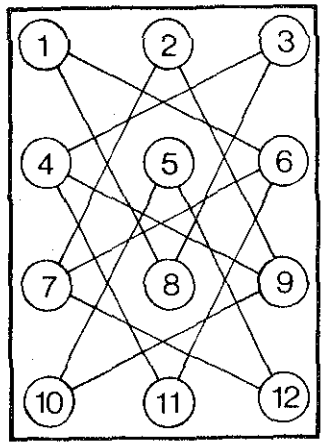
Rating: ** (*** in minimum moves).

E29

Titles and dates: **Fox and Geese**—from *The Canterbury puzzles* by H. E. Dudeney (1907 or earlier); **Intercity**—manufactured by Stratabord Ltd., UK (1976) (see Plate VIII); **Wizzard's Web Puzzle**, Canada (1981); **The Gagarin Puzzle**—one of a series under the general title of 'Magnetic Spaceman' (1984).

Start position: Three white counters (geese) on 1, 2, 3 and three black (foxes) on 10, 11, 12.

Object: Exchange the positions of the white and black counters, moving only along straight lines and stopping

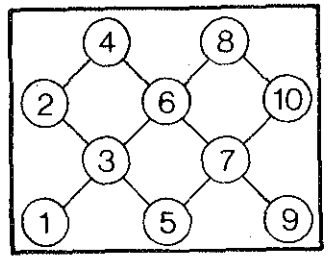


only on circles. At no time must fox and geese be at the ends of the same straight line. Each circle moved to counts as a move.

Solution: 22 moves.

Rating: **

E30



Titles and dates: **A New Counter Puzzle**—from *Amusements in mathematics* by H. E. Dudeney (1917 or earlier); **Manoeuvre**—manufactured by Stratabord Ltd., UK (1976) (see Plate VIII); **Zig-Zag Puzzle**, Canada (1981); **Venus Puzzle**—one of a series under the general title of 'Magnetic Spaceman' (1984).

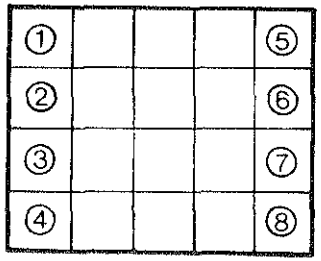
Start position: Two white counters on 1 and 2, two black on 9 & 10.

Object: Exchange the positions of the white and black counters, moving only along straight lines and stopping only on circles. At no time must counters of different colours be on the same straight line. Counters may move as far as they like in a straight line per move and may cross a line on which a different colour rests, but cannot stop there.

Solution: 18 moves.

Rating: ***

E31



Titles: **A New Bishop's Puzzle**—from *Amusements in mathematics* by H. E. Dudeney. Also **Interchange**—made by Four Generations USA, one of a series of 'Penny puzzlers'; **The Bishop's Puzzle**—one of a series under the general title of 'Magnetic Norman'.

Dates: 1917 or earlier (Dudeney); 1974 (Interchange); 1984 (Magnetic Norman).

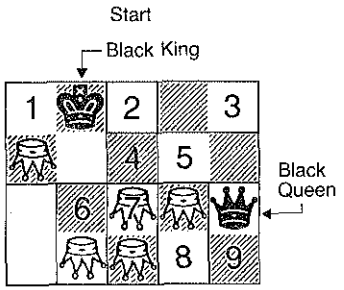
Start position: Four white counters (bishops) on 1, 2, 3, 4; four black on 5, 6, 7, 8.

Object: Exactly as previous puzzle (E30) except that pieces may move only diagonally and that counters of different colours must never be on the same straight diagonal lines.

Solution: 36 moves.

Rating: ***

E32



Title: **Sliding Checkless Chess**—invented by W. H. Reilly and published as Problem No. 8547 in *Fairy Chess Review*.

Date: November 1949.

Pieces: Represent a cut-up section of a chess board; on some pieces there are one or two chessmen. The chessmen can only be moved by sliding the individual sections of chess board until the required position is reached. The chessmen are: one black king, one black queen and five white 'grasshoppers', which are represented by upside down queens. For those unfamiliar with Fairy Chess, a grasshopper moves like a queen, but only to a square immediately beyond one man in the line. For the purposes of this puzzle, therefore, a grasshopper will only put the King into check if there is another man (of either colour) immediately in front of the King.

Start position: As shown in the diagram.

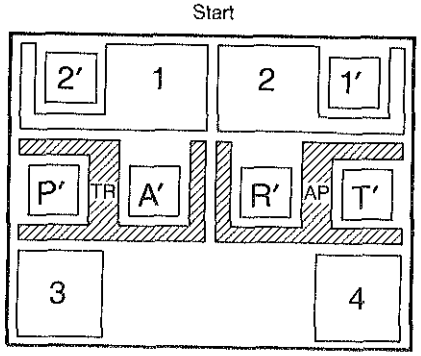
Object: White to give checkmate in 165 moves! Simple checks are ignored and, of course, no chessmen are removed from the board. For those who do not wish to solve the chess problem but would like to tackle the sliding block puzzle, the solutions section first gives the required 'final' position. Although this puzzle uses the same shaped pieces as puzzle C19 (turned on its side), it is a quite different (and more difficult) puzzle.

Solution: 165 moves (169). [Reilly's published solution had 240

moves and he said: 'this task is perhaps too formidable for general solving'!!]

Rating: ***

E33



Title: **Trap**—by Minoru Abe, Japan.

Date: 1986.

Pieces: Four domino-shaped pieces and eight square pieces. Six of the square pieces (P′, A′, R′, T′, 1′, 2′) are slightly smaller than the other two (3, 4) and fit into slots cut into the domino pieces. Squares 3, 4 cannot fit into the domino slots. Pieces 1 and 2 each have one slot and pieces A (marked A, P) and T (marked T, R) each have two slots. All the slots are cut into the side of the dominoes, except P and T which are cut into the end of the dominoes. The six smaller squares are only slightly smaller than the large ones and cannot pass each other in the space of a domino.

Start position: As shown in the diagram.

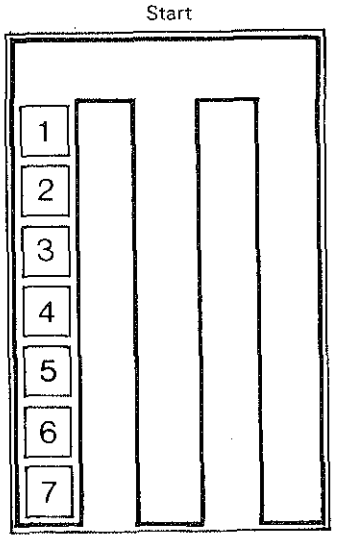
Object: Get the small squares—P′, A′, R′, T′, 1′, 2′—into their correct 'home' positions as marked on the domino pieces. The domino pieces should end up in their original positions so that the word TRAP is read across the centre. In other words, squares P′ and T′ must be exchanged, as well as A′ and R′, and 1′ and 2′.

Solution: 177 moves (196). The definition of a move is normal save that when a domino piece moves, so must the

small square(s) in its slot(s): this counts as one move—not two or three. (An insight on how to solve this type of puzzle is given in the solutions section.)

Rating: ****

E34



Titles and dates: Patented in USA (Patent No. 347 596) by A. B. Harris in 1886; also patented in UK (Patent No. 20 672) by A. Gartner and G. Talcott in 1890. **Die Vertrackte Schiebe**—manufactured by Quandt, West Germany, (1970s); **Chronologic**—manufactured by Mag-Nif Inc., USA (1975); also produced as one of a series of four 'Tricky Button Puzzles'—made in Hong Kong (1984); a further version recently made of cheap plastic in Russia is of unknown date.

Pieces: Tricky Button has five pieces; Chronologic has six pieces; Die Vertrackte Schiebe has seven pieces; the Russian version has eight pieces.

Start position: As shown in the diagram, which gives an example of a seven-piece puzzle.

Object: Move all the pieces to a different 'leg'. No piece can be moved to a leg that contains a lower number. Note that a piece may move any distance in one move.

Solutions: 31 moves (5 pieces)
63 moves (6 pieces)
127 moves (7 pieces)
255 moves (8 pieces).
A solution is given to the seven-piece puzzle.

Ratings: * to **

E35

Title: **Panex** (two versions: Gold and Silver)—invented by T. Akanuma and manufactured by Tricks Co. Ltd., Japan.

Date: 1983.

Pieces: Twenty pieces in two columns of ten in the left and right 'legs'—see the diagram for the previous puzzle (E34). They fit into grooves into the sides of the tracks and therefore can not be removed. Because of the (hidden) shape of the grooves and the (hidden) projections on the backs of the pieces, the topmost piece in each column can only descend to the topmost position in any of the three legs. Similarly, the pieces second from the top can only descend to the position second from the top in any leg, etc. There are ten pieces of one colour and ten of another. In the Gold version, the ten pieces of each colour all look the same; but in the Silver version, there is a pyramid drawn across all ten pieces which will indicate to what level a piece will descend down a leg. There is no difference in the solving of Gold or Silver—only that the Gold version is harder because all the pieces in a column appear to be identical.

Start position: As shown in the diagram for the previous puzzle (E34) except that there are two columns each of ten pieces—in the left and right hand legs.

Object: (a) exchange the two columns of pieces.
(b) move one column of pieces to the centre leg.

Solutions: Require thousands of moves! Which puzzle requires the fewest moves and what is the minimum number of moves needed to solve them? See solutions section.

Rating: ******

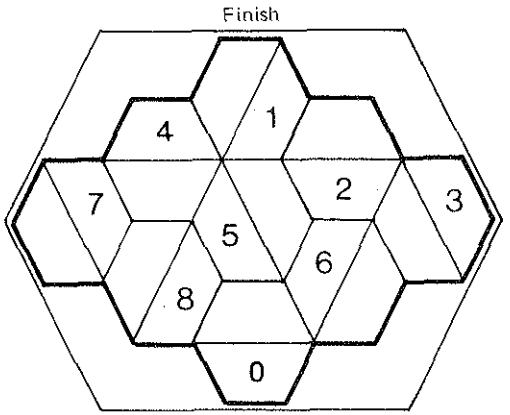
8

SLIDING BLOCK PUZZLES WITH SPECIAL SHAPED PIECES OR WHERE ROTATION IS ALLOWED, F1–F13

Two different categories are included in this section: puzzles with odd shaped pieces (F1–F4), and puzzles in which rotation of the pieces is allowed (F5–F13). In most sliding block puzzles turning the pieces is strictly not allowed, but there are some half dozen or so exceptions, which are given here. They are all very easy except for the last three (F10, F11, F12), which deserve special mention. They were invented by Shirley Ellis Stotts (blind from the age of six), who came up with the idea of building a 'fence' around a particular piece once it had reached its destination and also of allowing the rotation of rectangular pieces. For an explanation of what constitutes a move, refer to the 'moves' section in Chapter 1 (see page 5).

The better puzzles are F1, F4, F10, F11, F12, F13.

F1



Title: **Nine-Hex Puzzle**—manufactured by Clement Toy Co., USA.

Date: *ca.* 1922–4.

Pieces: Note that each piece is half a hexagon. There are 16 pieces: eight numbered half-hexagons and eight unnumbered half-hexagons.

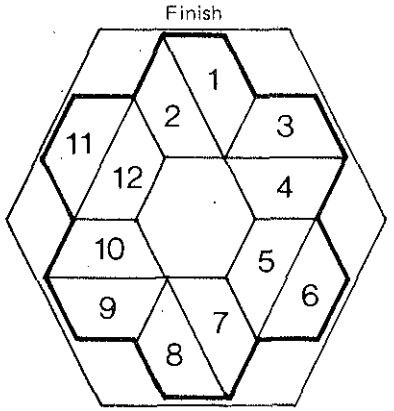
Start positions: Some three or four dozen specific problems were given with the instructions to this puzzle. Only one (of the most difficult) is given. Start—807, 654, 321 where 8 is in the 1 position, 0 (space) in the 2 position, 7 in the 3 position, etc.

Object: Finish as shown. No rotation of pieces allowed.

Solution: 81 moves.

Rating: ****

F2



Title: None—by A. C. Crehore, Cleveland, Ohio.

Date: Patented in USA in 1924 (Patent No. 1 495 576) by A. C. Crehore.

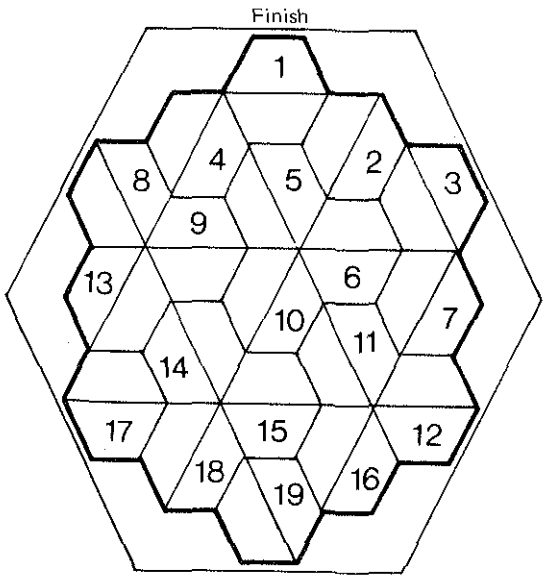
Pieces: Note that each piece is half a hexagon.

Start position: Random. Remove complete centre hexagon.

Object: Finish as shown. No rotation of pieces allowed.

Rating: ** or ***

F3



Title: None—by A. C. Crehore, Cleveland, Ohio.

Date: Patented in USA in 1924 (Patent No. 1 495 576) by A. C. Crehore.

Pieces: Note that each piece is a half-hexagon.

Start position: Random. Remove any complete hexagon (two pieces) e.g. piece 19 and its blank hexagon counterpart.

Object: Finish as shown. No rotation of pieces allowed.

Rating: *** or ****

F4

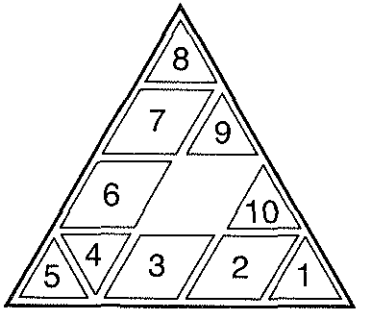
Title: Unknown—by T. R. Dawson in *Original Puzzles* (manuscript—but may have been published elsewhere).

Date: December 1928.

Pieces: Four diamond- and six triangular-shaped pieces.

Start position: As shown in the diagram.

Object: Exchange the positions of the following pieces: 1 with 8, 2 with 7, 3 with 6, and 9 with 10. 4 and 5 should end up where they start. The numbers will end up reading



anticlockwise round the triangle. No rotation of the pieces is allowed.

Solution: 68 moves.

Rating: **

F5

Title: **Get Protection into Great Britain and turn out Free Trade** (see Plate VIII).

Date: *ca.* 1903.

Pieces: Two quarter-moon shaped pieces, one slightly larger than the other.

Start position: Refer to Plate VIII—gives start positions for the two pieces.

Object: Move the larger of the two pieces to the place marked 'United Kingdom of Great Britain and Ireland'. The other should be moved away from its START position. The pieces may only move within the sunken channel and must not be lifted out.

Solution: The problem is how to make the two pieces pass each other whilst remaining in the channel.

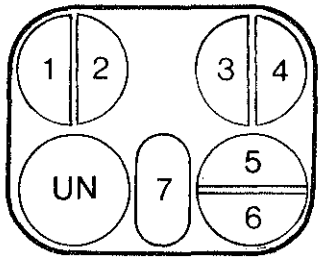
Rating: *

F6

Title: **Beat the Axis**—from *Industrial Arts and Vocational Education Magazine*, USA.

Date: September 1944.

Start

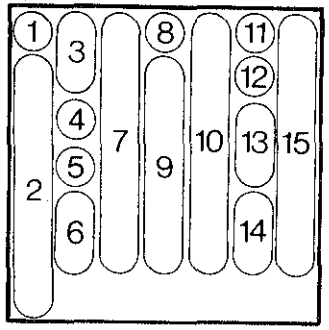


Pieces: Bottom left corner is marked Allies 1, bottom right corner is marked Axis 2.
Start position: As shown in the diagram.
Object: Move UN to bottom right corner. Note: rotation of pieces is allowed.
Solution: 17 moves.
Rating: *

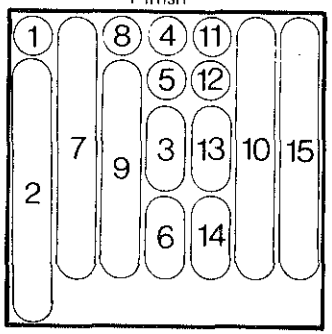
F7, F8

Title: None.
Date: Patented in USA in 1906 (Patent No. 811 321) by E. C. Pfeiffer.
Start positions: As shown in the diagrams.
Object: F7—finish as shown. Note: rotation of pieces is allowed.

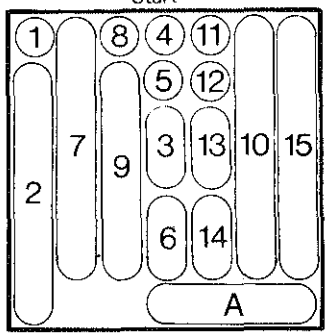
Start



Finish



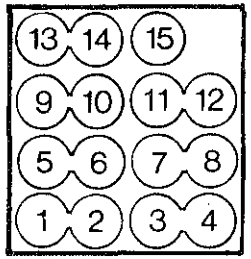
Start



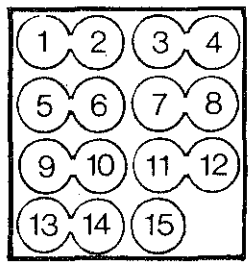
F8—move A to bottom left corner. Note: rotation of pieces is allowed.
Solutions: F7—24 moves.
F8—13 moves.
Rating: *

F9

Start



Finish

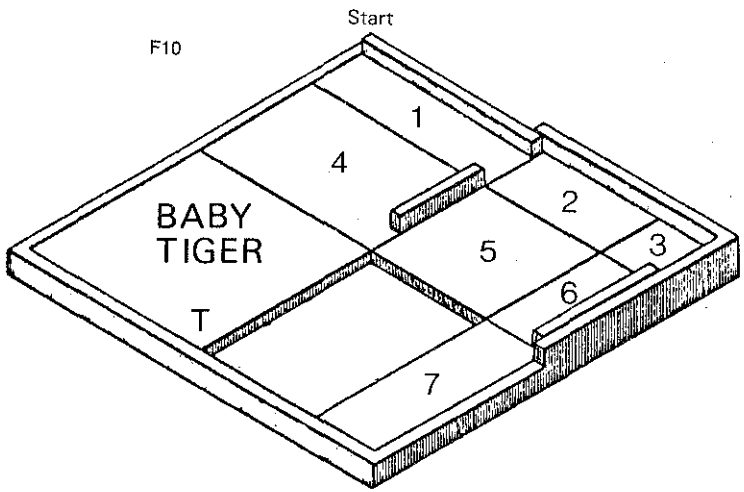


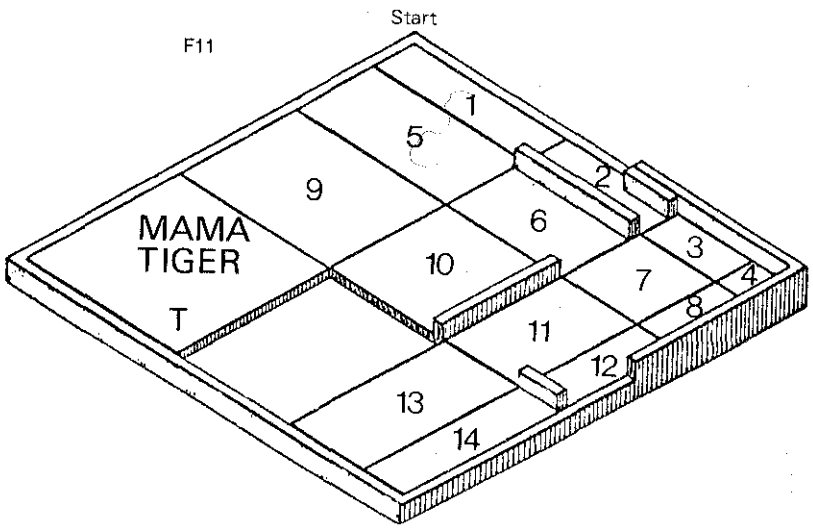
Title: None—by F. G. Dustin, Minneapolis.
Date: Patented in USA in 1935 (Patent No. 1 993 211) by F. G. Dustin.
Pieces: Should fit fairly tightly into surrounding box.
Start position: As shown in the diagram.
Object: Finish as shown in the diagram. Note: rotation of pieces is allowed.
Solutions: Several are possible, and they depend on how tightly

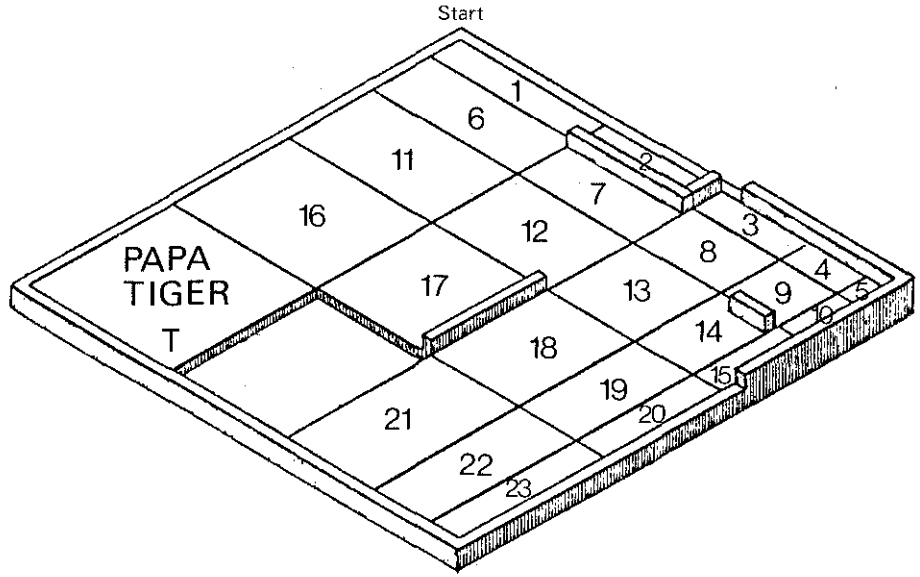
the pieces are made to fit the box. For this reason the solution is left to the reader.

Rating: **

F10–F12







Titles: **Baby Tiger Puzzle; Mama Tiger Puzzle; Papa Tiger Puzzle**—by S. E. Stotts, Denver, USA. Also **Wooden Puzzle**—made by Hanabishi, Japan.

Date: Patented in USA in 1965 (Patent No. 3 208 753) by S. E. Stotts.

Pieces: Should fit fairly tightly in surrounding box. The sizes of the pieces are given by the square pieces, which are 1 unit × 1 unit, 2 × 2, 3 × 3, and for the larger puzzles 4 × 4 and 5 × 5. The sizes of the other pieces will then be obvious.

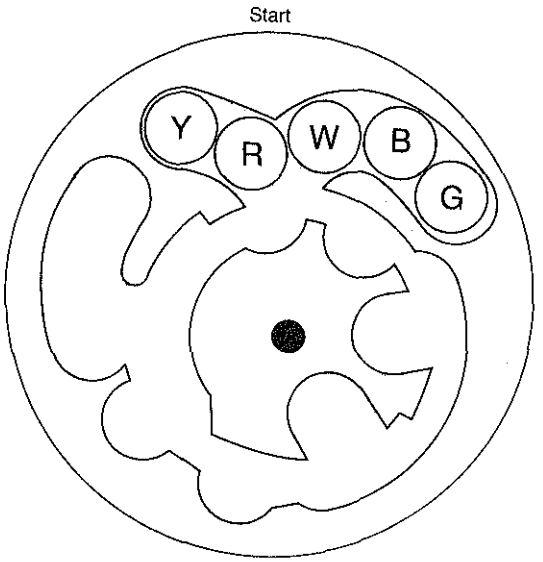
Start positions: As shown in the diagrams. In F10 it is necessary to pick up piece 7, rotate it 90° and place it underneath 5, 6 before starting.

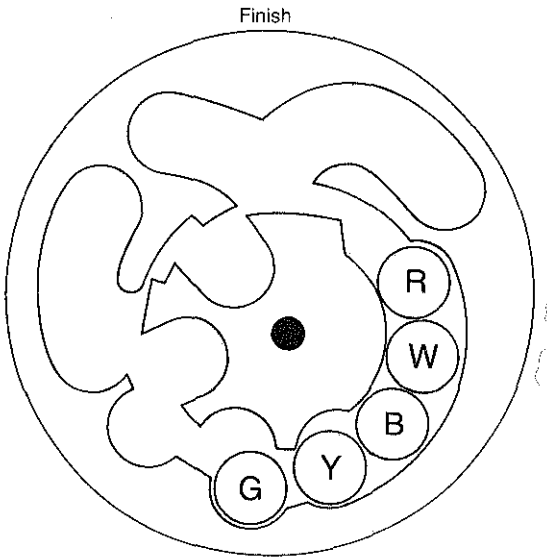
Object: Move T (tiger) to top right corner and surround completely with pieces with raised edges (fence). Note: rotation of pieces is allowed.

Solutions: F10—49 moves
F11—131 moves
F12—245 moves.

Ratings: **F10; ***F11; ****F12.

F13





Title: **The Combination Lock Puzzle**—manufactured by Combination Novelty Co., USA.

Date: *ca.* 1895. Patented in 1896 by W. E. Dow (US Patent No. 560 197). The puzzle is made of plastic and this must represent one of the first uses of the material. The plastic-making process had evidently not been perfected, as the plastic is quite brittle and chips easily.

Description: A black disc with a sunken area in which there are five coloured plastic counters. The counters each have a letter denoting their colour: Y (yellow), R (red), W (white), B (blue), G (green). Near the centre of the puzzle is what looks like an island, but is really a 'rotor', which is free to rotate on a spindle. The rotor will rotate only through about 330°, as a lip on its furthermost promontory comes into contact with another lip adjacent to the red counter when in its start position.

Start position: As shown in the illustration. The counters should be in the slot at the top in the following order (reading clockwise): Y R W B G.

Object: Move the counters to the positions shown in the 'Finish' illustration. Counters should be in the bottom right quadrant of the puzzle in the following order (reading clockwise): R W B Y G. Counters may only be slid along the channels, using the rotor where appropriate. When turned fully clockwise, the rotor will not accept a counter from the starting position into its largest 'bay'.

Solution: No set number of moves is given, as it is so difficult to determine exactly what a move consists of. However, a step by step solution is given in the solutions section.

Rating: ***

9

THREE-DIMENSIONAL SLIDING BLOCK PUZZLES, G1–G5

Three-dimensional sliding block puzzles were proposed theoretically a hundred years ago (Tait 1880; patent by Rice 1889), but it was not until after World War II that the mechanical problem of containing the blocks was solved.

Many puzzles of this type are simply three-dimensional versions of the '15' puzzle, often with colours instead of numbers. The problem is to mix up the pieces and then put them back into their correct positions. Plate X shows a selection and, as they are mostly very easy to solve, they are not described here.

A few of the more interesting ones are shown in Plate IX and are discussed in this chapter. G1 is a particularly interesting puzzle containing two layers of six and five blocks (tablets). Blocks can be slid in the normal way, in addition to passing from one layer to the other. A curiosity rather than a puzzle is G5—a box which is (un)locked by sliding various sections (levers) of wood.

G1, G3, and G4 are the more interesting of this category.

G1

Title: **Change the Seasons**—invented by Arthur L. Smith, published in *Popular Science Magazine*, USA (see Plate IX).

Date: July 1940.

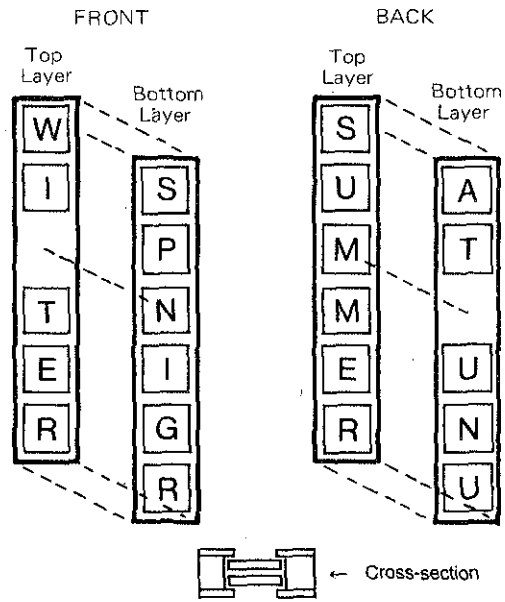
Pieces: Eleven blocks in two layers, free to slide in an enclosure with the blocks visible from front and back. Blocks have letters on both sides as follows: WA, IT, TU, EN, RU, SS, UP, MN, MI, EG, RR.

Start position: As shown in the diagrams—front reads WINTER, back reads SUMMER.

Object: Change WINTER to read SPRING (front); at the same time change SUMMER to read AUTUMN (back).

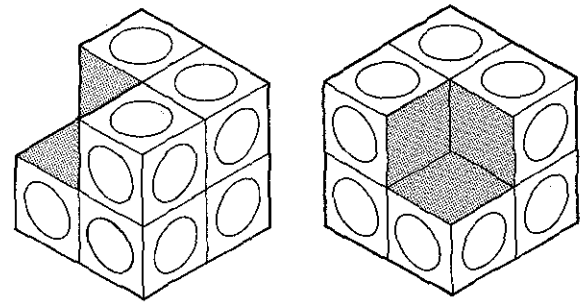
Solution: 51 moves.

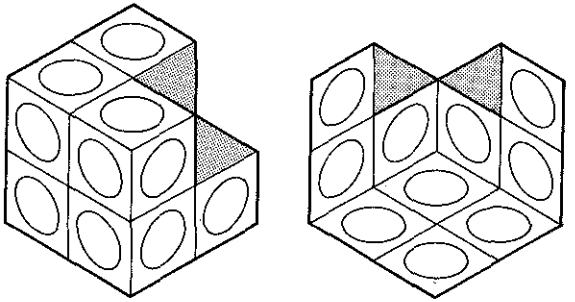
Rating: **



G2

Title: **Bloxbox; Qrazy Qube**—by P. Hein, Denmark (see Plate IX).





Dates: Patented in 1889 by C. I. Rice (USA Patent No. 416 344); 1973 (Hein); patented again in 1974 (US Patent No. 3 841 638) by F. W. Sinden.

Pieces: Seven small cubes in one large transparent cube (not shown), which has space for eight cubes. Each small cube is identical, having three large spots on three faces which meet at a corner, and three blank faces which meet at the opposite corner. The cubes are packed so that it is possible to have all 21 spots on the outside of the cube—that is, touching the inside faces of the large transparent cube.

Start position: As shown in the diagram.

Object: Get all 21 blank faces on the outside, that is, touching the inside faces of the largest transparent cube. Blocks are moved by tilting the large cube so that the small cubes slide by gravity. An alternative puzzle is to mix up the blocks at random and then arrange the blocks in the start position again as shown.

Solution: Impossible to get all blank faces on the outside. However, a general procedure for getting the puzzle back to its original position from a mixed up state is given in the solutions section.

G3

Titles: **Varikon Box 'L** (Hungary); **Mad Marbles; Inversion** —manufactured by Toys & Games International, USA (see Plate IX).

Date: Patented in 1905 (US Patent No. 785 665) by A. Coe;

also patented in 1974 (US Patent No. 3 845 959) by D. Kosarek. The versions mentioned above were all manufactured in the 1980s.

Pieces: This puzzle is a 3 × 3 × 3 version of the previous one (G2). The centre piece on each face is either fixed (Varikon Box 'L) or absent (Inversion). In Mad Marbles there is a centre column around which all the marbles can freely move.

Start position: All one colour on the external faces (Varikon, Inversion); random (Marbles).

Object: Change the colour on all the external faces.

Solution: Possible—see solutions section.

Rating: ***

G4

Title: **Pionir Cube**—made by Artex, Hungary (see Plate IX).

Date: 1984.

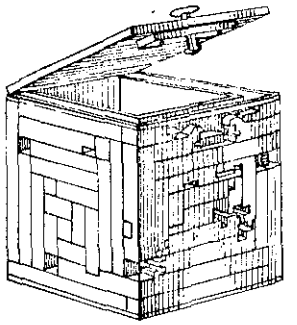
Pieces: 67 beads inside a transparent cube. There are 4 red, 4 yellow, 4 green and 55 black beads. These beads are along all the edges of the cube with one space.

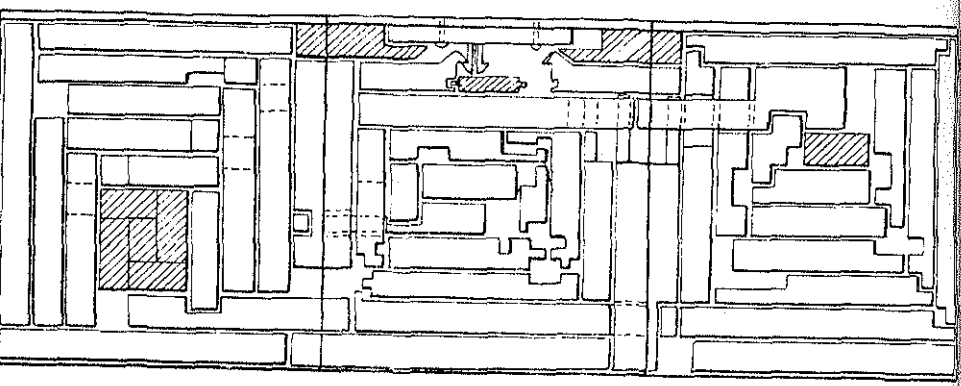
Start position: Random

Object: Put the 12 coloured beads into their correct positions. These are at the centre of each edge, each set of one colour forming a ring around the cube.

Rating: ***

G5





Title: None—by O. T. Marsh, Nashville, Tennesse.

Date: Patented in USA in 1965 (Patent No. 3 216 558) by O. T. Marsh.

Pieces: Approximately 65 blocks free to slide (in a certain sequence) but fixed to three sides of a box.

Start position: As shown in the diagram.

Object: Slide the blocks so as to free the catch, which lets the box be opened. A smaller version with fewer blocks is also shown in the same patent.

Comment: This is not so much a sliding block puzzle as a trick opening box. However, since the opening mechanism works exactly on a sliding block principle, it is included here as a curiosity.

10

SOKO (WAREHOUSE) PUZZLES, H1–H10

An entirely new class of sliding piece puzzle has recently been invented (1983?) in Osaka, Japan by a company whose name, in English, means 'Thinking Rabbit'. Most of these puzzles were designed as computer games, with secret walls, hidden compartments etc., and do not fall within the meaning of a sliding piece puzzle. However, the first ten designs that appeared are a half-way house between sliding block and railway shunting puzzles. While all the pieces in a sliding block puzzle are capable of independent movement, either singly or in groups, the majority of the pieces in a railway shunting puzzle must be pushed or pulled. In this Japanese variation, there is only one piece that can move the others, and it can only push.

These puzzles are called 'Soko' or 'Sokoban' puzzles. Soko is the Japanese word for warehouse and Sokoban means warehouseman or storeman. The story goes as follows. A storeman is left in charge of an underground(?) warehouse, which consists of a number of rooms or chambers connected by doorways and corridors. Inside these rooms various packing cases have been left lying around and the storeman has been told that he must get them all into one particular chamber. The packing cases are very heavy and have no handles. The storeman is just strong enough to push them, but only one at a time; he cannot pull them. In order to push a packing case, the storeman needs to be able to get completely behind it. He cannot rotate them by pushing on their edges and then slip in behind. Obviously, if a packing case is pushed into a corner, it is stuck there.

The object of the exercise is to get all the packing cases—represented by numbered squares—into the positions marked X in one of the chambers. For convenience it is assumed that the storeman is the same size as a packing case and that he cannot jump over a packing case, nor squeeze between them unless there is at least the area of a packing case (= one square) available. He must have a whole square behind a packing case to be able to push it from square to square.

In solving these puzzles, it is only necessary to record the movement of the numbered pieces (packing cases), as it will be obvious how the storeman has pushed them. The movement of the storeman is therefore 'free' and the number of moves relates only to the numbered pieces.

There are two possible definitions of a move:

(i) Push one piece in any direction or combination of directions; moves should all be orthogonal and without rotation.

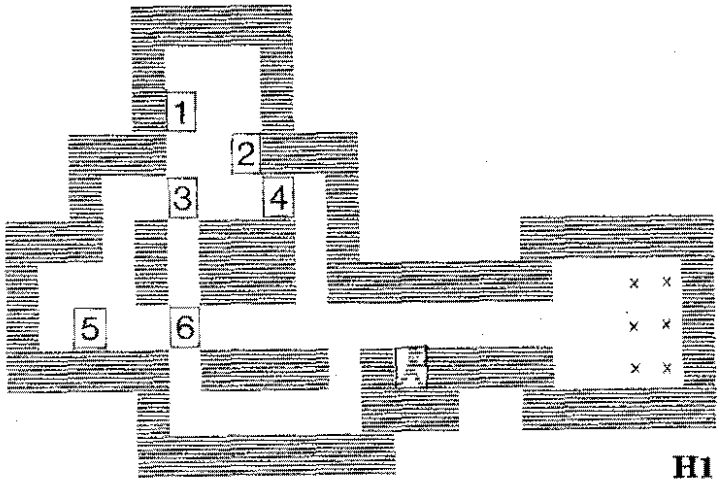
(ii) Push one piece in any one direction; moves should all be orthogonal and without rotation.

As with the sliding block puzzles, the number of moves for both the above types is given, the ones for definition (ii) being in brackets.

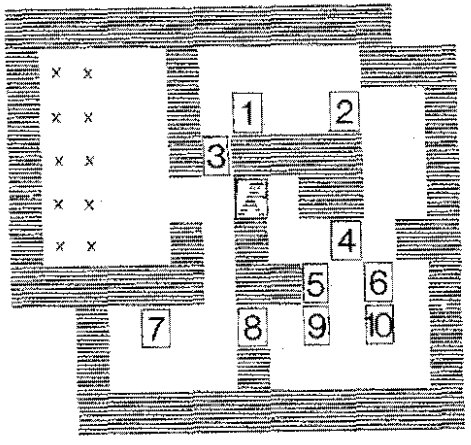
The number of moves required and the difficulty rating for each puzzle is given below:

H1	9 moves	(22)	*
H2	16 moves	(53)	**
H3	16 moves	(43)	**
H4	23 moves	(106)	***
H5	14 moves	(45)	**
H6	14 moves	(38)	***
H7	18 moves	(37]	***
H8	23 moves	(86)	****
H9	20 moves	(72)	****
H10	44 moves	(198)	*****

The start position of the storeman is shown by a little man.

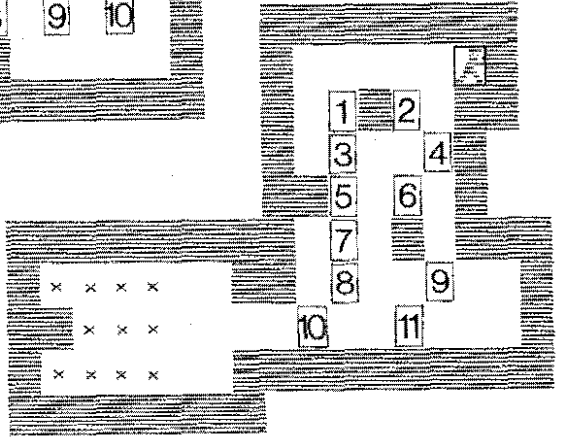


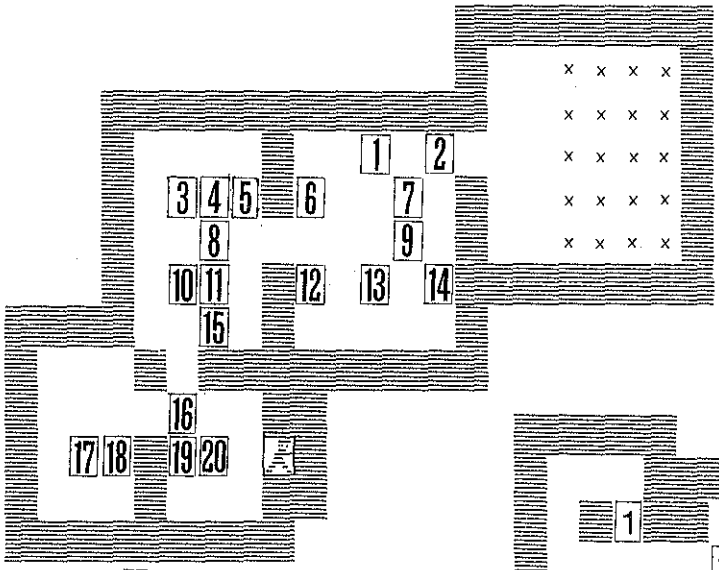
H1



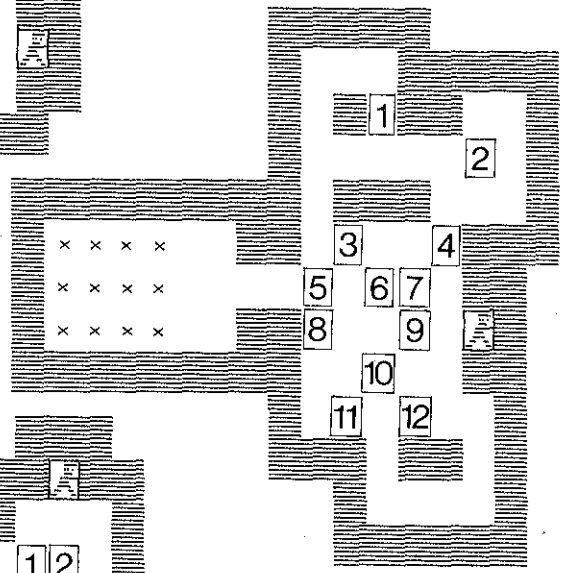
H2

H3

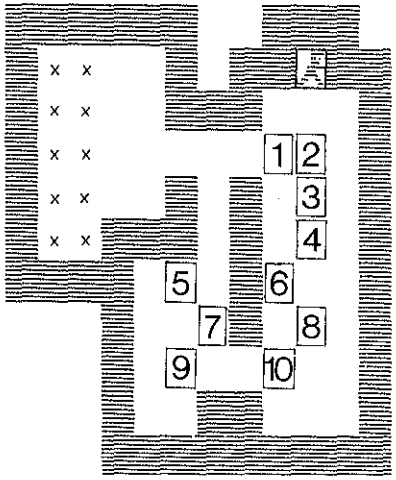




H4

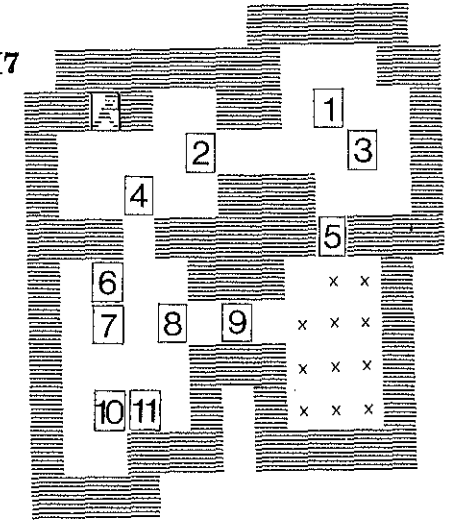


H5

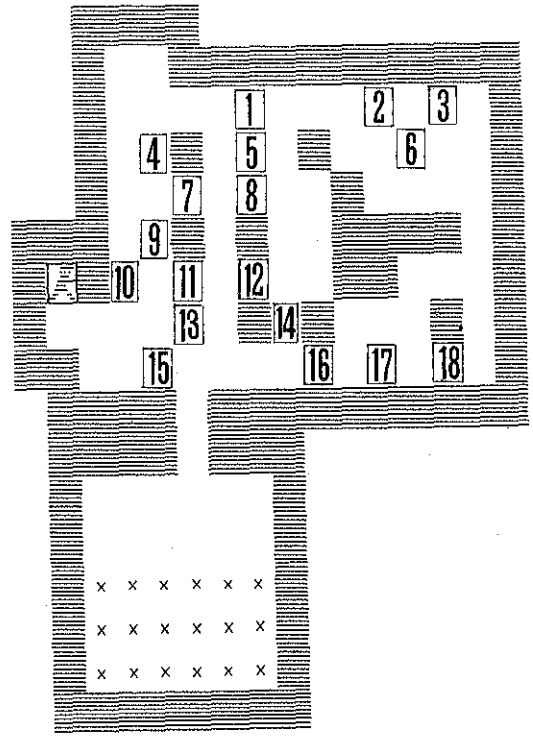


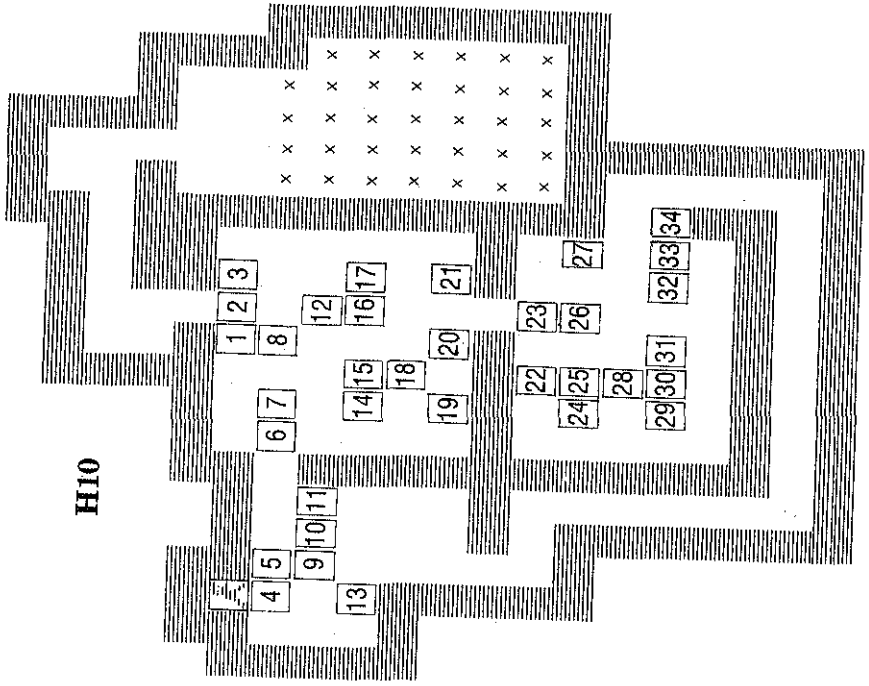
H6

H7

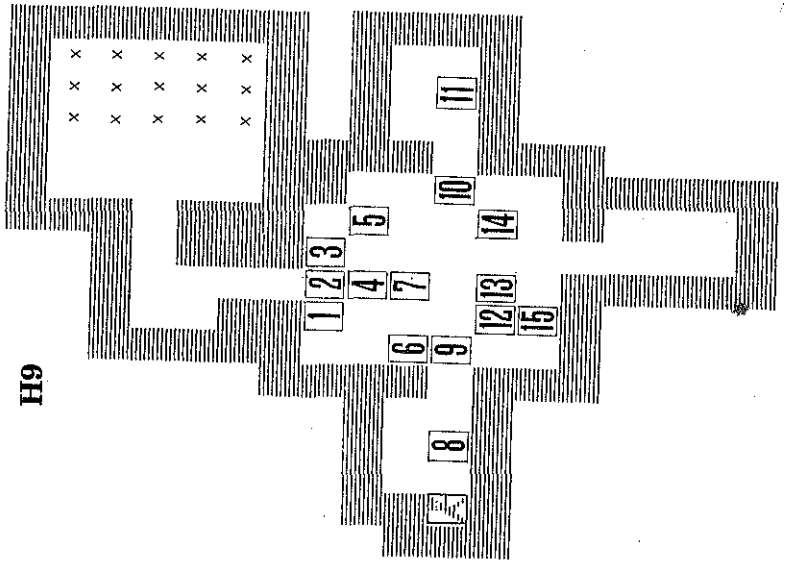


H8





H10



H9

11

RAILWAY SHUNTING PUZZLES, J1–J13

Railway shunting puzzles are a class of sliding piece puzzle where one or two pieces (engines or locomotives) must push or pull the other pieces (cars, coaches or wagons) into predetermined positions. Typically, two trains on a single track must pass each other using a siding or loop branch line, which can only take a certain number of wagons and sometimes may not take the engine. Another type of shunting puzzle has a turntable and the aim is either to turn a whole train around or to get two trains past each other.

Railway shunting puzzles go back many years—almost to the same date as the original '15' puzzle. The first US patent was issued to P. Protheroe in 1885 (No. 332 211), and was of the turntable variety. The object was to mix up the blocks at random and then, using only the engine and turntable, to arrange the pieces (wagons) so that the letters on them spelt 'Humpty Dumptie'. The second patent was in 1890, but Ahrens (1892) says that he purchased the puzzle in 1883 or 1884. It has been by far the most popular shunting puzzle and has been reproduced time and again in books and magazines right up to the present day. The patent was granted to A. G. Farwell (US Patent No. 437 186) and proposed the triangular branch line principle. The puzzle looks so simple—having only three pieces—but is rarely solved at the first attempt (see J1). The puzzle was very popular and it is known that a version was actually made and sold in the 1890s (Dudeney 1925), although no original model has yet come to light. At least six other patents for the same or very similar puzzles were issued between 1892 and 1921.

Although railway shunting puzzles make good puzzles, they have never been produced in the quantity or variety that sliding block puzzles have. This is probably due to the difficulty of manufacture and their high cost of mass production. A sliding block puzzle only needs a tray with (wooden) blocks that are square or rectangular; the shunting puzzle needs some kind of groove—often

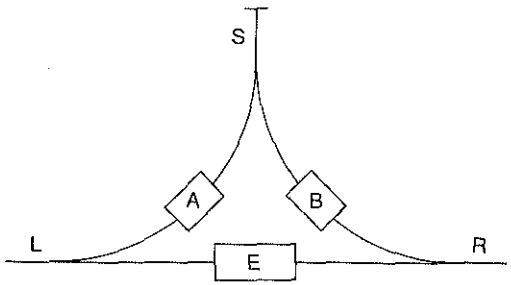
curved—and sometimes a turntable as well. Until fairly recently manufacturing techniques have not permitted these kinds of operations on a cheap enough basis.

There seem to be several dozen different puzzles, but many of them are of no special interest or merit, or are hopelessly complicated. Therefore, only the most worthwhile have been included.

No tricks, such as flying switches or uncoupling cars whilst in motion, are employed. Except where otherwise stated, engines have couplings both front and back and can therefore pull or push in both directions. A move is defined as a change in the direction of an engine except in the case of puzzles employing a turntable, when it is its rotation that counts as a move.

Pieces from the envelope at the back of the book can be used for solving the puzzles, and it will only be necessary to draw the outline of the puzzles on paper or card with a felt tip pen.

J1



Titles and dates: **Great Northern Puzzle** (1883/4); **Switching Cars**—from *Sam Loyd and his puzzles* by S. Loyd (1928); also from *Avec des Nombres et des Lignes* by A. Sainte-Laguë (1942); **Keeping out of a Trap**—from *Maths in fun* by Joseph Degrazia (1949).

W. W. Rouse Ball (1892) says that he purchased a Great Northern Puzzle in 1883 or 1884. It is the most widely known railway shunting puzzle and has maintained its popularity since its inception.

The first patent does not seem to have been issued until 1890 (Arthur G. Farwell: US Patent No. 437 186), but a whole range of patents dealing with the same or a similar design continued to be granted until 1921. These are: No. 482 957 by J. Allen (1892); No. 688 339 by J. M. Rodgers (1901); No. 703 076 by F. L. Napier (1902); No. 822 862 by W. E. McGraw and A. M. Goodale (1906); No. 1 275 210 by J. M. Braunschweiler (1918); No. 1 377 039 by J. V. Wells (1921).

Description: An engine E is on a main track LR. Two wagons A and B are on two short tracks that lead to a siding S. The siding can accommodate either of the wagons but not the engine, which is too long.

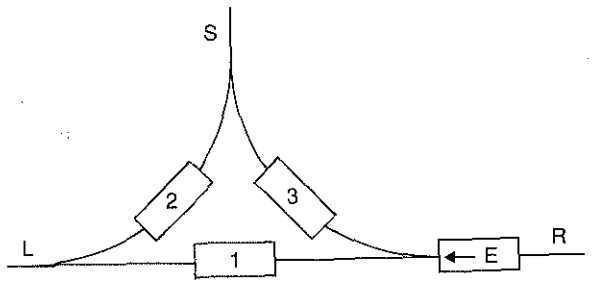
Start position: As shown in the diagram.

Object: Exchange the positions of A and B while leaving E on the main track LR, in as few moves as possible. Every time the engine changes direction, it counts as a move.

Solution: 16 moves.

Rating: **

J2



Title: **Celluloid Starch Puzzle**—from a printed card used as advertising—USA.

Date: *ca.* 1900–1910.

Description: An engine (E) is on track R and three cars (1, 2, 3) are on the tracks between the switches at S, L, R. The engine is facing towards L.

Start position: As shown in the diagram.

Object: To turn the engine around—leaving it on track R and all three cars in their original positions in as few

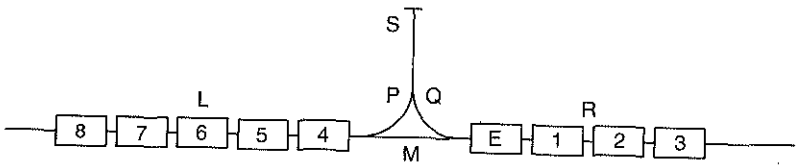
moves as possible. Each time the engine changes direction, it counts as a move.

Solution: 10 moves.

Comment: A slightly harder problem is to assume that the siding S can only accommodate either one car or the engine at a time (solution—14 moves).

Rating: **

J3



Titles: **The Switch Problem**—from *Cyclopedia of 5000 puzzles, tricks and conundrums* by S. Loyd; **Pacific 231**—manufactured by Atelier des Alpes, France (see Plate XII).

Dates: 1914 or earlier (Loyd); 1970s ('Pacific 231').

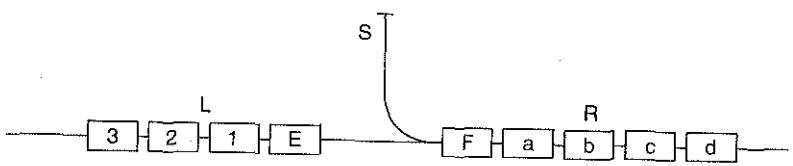
Description: A train consisting of an engine and three wagons (E, 1, 2, 3) is on line R. Another train with a broken-down engine (4, 5, 6, 7, 8) is on line L. The siding (S) and branch lines (P, Q) can each take one wagon or engine.

Start position: As shown in the diagram.

Object: Get the trains past each other. The broken-down engine (4) is to be treated as just another wagon.

Rating: **

J4



Titles and dates: **Primitive Railroading**—from *Cyclopedia of 5000 puzzles, tricks and condundrums* by S. Loyd (1914); **The Short Siding**—from *Maths is fun* by Joseph Degrazia (1949); **Whistle Stop Puzzle**—manufactured by Pentangle, UK, 1970s (see Plate XII).

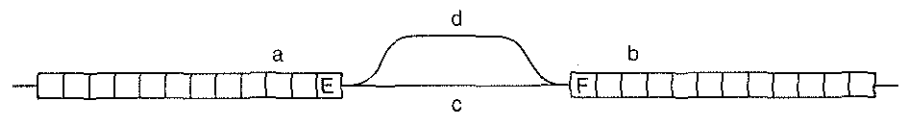
Description: Two trains—1, 2, 3 pulled by engine E and a, b, c, d pulled by engine F—are on the main line track L, R. The only passing place is siding S which is large enough to take only one car or one engine. The engines have no couplers in front and therefore cannot pull any cars while going backwards.

Start position: As shown in the diagram.

Object: Make the trains pass each other on track L R with as few movements as possible.

Rating: **

J5



Titles and dates: **Shunting Problem at a Station**—from *Maths is fun* by Joseph Degrazia (1949); **Alfred croise Éloi**—from *Avec des nombres et des lignes* by A. Sainte-Laguë (1942).

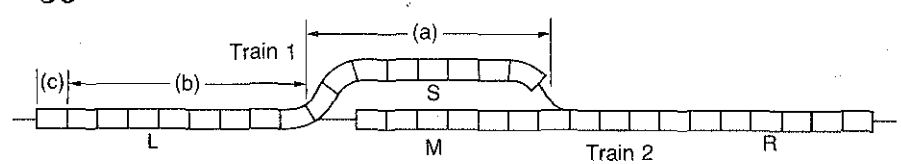
Description: A train E is on a track at a and another train F is at b. Each train has 12 coaches (including the engine). The main track c and loop branch line d can each accommodate eight coaches.

Start position: As shown in the diagram.

Object: Get the trains past each other so that train E is at b and train F is at a.

Rating: *

J6



Title: **A Railway Muddle**—from *Amusements in mathematics* by Henry E. Dudeney.

Date: 1917 or earlier.

Description: Two trains (1 and 2); each consisting of an engine and 16 wagons meet at a branch line loop. The loop branch line (S) and the main line between the switches can each accommodate eight wagons (or an engine and seven wagons).

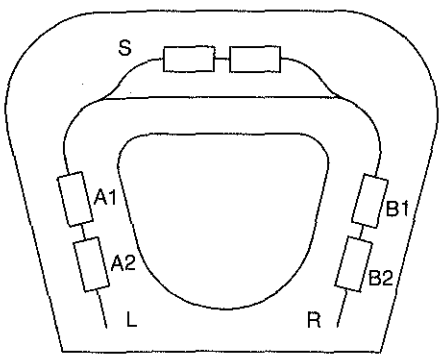
Start position: As shown in the diagram.

Object: Make trains 1 and 2 pass each other in as few moves as possible. Each time an engine changes direction it counts as a move.

Solution: 6 moves.

Rating: **

J7



Title: **Good Luck Railroad Puzzle Game**—made by Saleable Products Co., USA (see Plate XII).

Date: 1945.

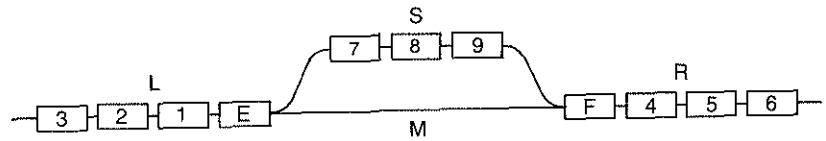
Description: A plastic horseshoe with a groove in which various coloured railway rolling stock is free to slide. Train A1 on the left (L) consists of an engine (A1) and a coach (A2) as does the train at R (B1 and B2). In the loop branch line, S, there are two freight wagons which have lost their couplings and can therefore only be pushed. The loop branch line can only take a maximum of three items.

Start position: As shown in the diagram.

Object: Make A and B pass each other, leaving the freight wagons in the siding S.

Rating: *

J8



Title: None—patented by L. B. Pennell (US Patent No. 783 589).

Date: 1905.

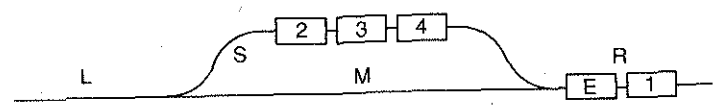
Description: An engine and three coaches (E, 1, 2, 3) are on a track LMR at L; and a further engine and three coaches (F, 4, 5, 6) are at R. On a loop branch line (S) there are three more coaches (7, 8, 9) that have no couplings and can therefore only be pushed—not pulled. The coaches 1, 2, 3 are permanently joined together and cannot be separated, as is the case with 4, 5, 6. The branch line and the main line between the switches will each take three coaches and an engine.

Start position: As shown in the diagram.

Object: Make the trains E, 1, 2, 3 and F, 4, 5, 6 pass each other, leaving 7, 8, 9 in the branch line S.

Rating: **

J9



Title: None—patented by J. W. Clark (US Patent No. 898 321).

Date: 1908.

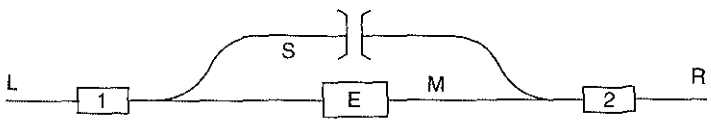
Description: An engine (E) and a wagon (1) are on a railway track at R. On a loop branch line (S) there are three wagons (2, 3, 4). The engine is old and small and the wagons are heavily laden; and so the engine can only push or pull one wagon at a time.

Start position: As shown in the diagram.

Object: Exchange the positions of wagons 1 and 3 and return the engine and other wagons to their original places.

Rating: ***

J10



Title: Unknown—from the *Academy of Recreational Mathematics Magazine*, Japan.

Date: 1983.

Description: Two wagons (1 and 2) and an engine (E) between them are on a main railway line, LR, which has a loop branch line, S. The branch line has a low bridge which the wagons can pass beneath but the engine cannot. It is possible for the engine to park a wagon underneath the bridge and to go round and pick it up from the other side.

Start position: As shown in the diagram.

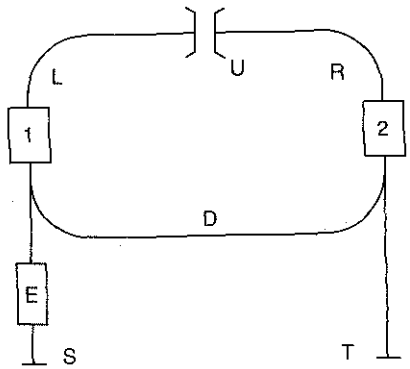
Object: Exchange the positions of wagons 1 and 2, leaving the engine at M.

Rating: **

J11

Titles and dates: Patented by L. J. White (US Patent No. 3 127 175) in 1964; **Aiguillages**—manufactured by Atelier des Alpes, France, 1970s (see Plate XII).

Description: A circular track has two wagons: 1 (at L) and 2 (at R). The wagons are piled high with freight and cannot



pass through the tunnel at U. There are two sidings: S and T. The engine house at S can only receive the engine, E, but no wagons. The engine can pass through the tunnel and both engine and wagons can enter siding T.

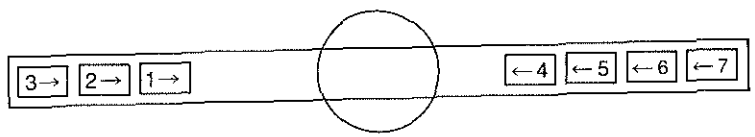
Start position: As shown in the diagram.

Object: Exchange the positions of the wagons, 1 and 2, and leave the engine E in its original siding, in as few moves as possible. Each time the engine changes direction it counts as a move.

Solution: 13 moves.

Rating: **

J12



Titles and dates: **Great Railroad Puzzle**—postcard puzzle produced by Jeffery and Son, Syracuse, New York in 1888; patented by O. L. Hubbard (US Patent No. 753 266) in 1904.

Description: Hubbard's patent proposes the puzzle as shown in the above diagram. Two trains, consisting of an engine (1) and two coaches (2, 3) and a further engine (4) and

three coaches (5, 6, 7) are facing each other on either side of a turntable. The lines on either side of the turntable can each hold a maximum of five coaches (engines being the same size as coaches). The turntable can only carry two coaches. The 'Great railroad puzzle' differs from the patent in that the lines on either side of the turntable are of unlimited length and there are an engine and four coaches on each side. Furthermore the coaches are not numbered and only the engines have to face the correct direction when the puzzle is solved. The 'Great railroad puzzle' postcard offered a prize of US $5.00, not for the shortest solution, but for a solution 'written in the least number of words'! Apart from its early date, it is otherwise not a very interesting puzzle. Hubbard's patent makes a far more satisfactory puzzle—this is the one given here.

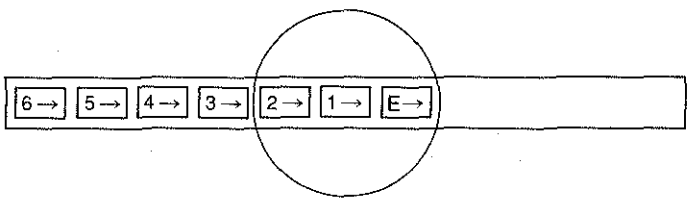
Start position: As shown in the diagram.

Object: Make the two trains pass each other, ensuring that both engines and all carriages face in the same direction as when they started. In other words, piece 1 will go to the place occupied by piece 7 (and face to the right), piece 4 will go to the place occupied by piece 3 (and face to the left) etc. Each time the turntable is rotated it counts as a move.

Solution: 16 moves.

Rating: **

J13



Titles and dates: **Shunting**!—manufactured by J. W. B. *ca.* 1900 (see Plate XII); **Turntable Train**—made by Pentangle, 1970s (see Plate XII).

Description: An engine (E) and six coaches (1–6) on a track and turntable. The engine and first two coaches are on the turntable, while the other four coaches are on the track. There is only room for four coaches on either side of the turntable. The turntable can accommodate two coaches plus the engine, which cannot leave it.

Start position: As shown in the diagram.

Object: Turn the train around so that the engine is facing left in front of the rest of the coaches, which should be in numbered order (2–6) extending to the right. Each time the turntable is rotated, it counts as a move. It is not difficult to achieve this with unlimited moves. Therefore the object is to find the minimum move solution.

Comment: The diagram above is for the Shunting puzzle. The Turntable Train puzzle is different in that the track is not in a straight line but is more or less at right angles. This means that extra moves will be needed to solve it.

Solutions: 19 moves (Shunting)
33 moves (Turntable Train).

Rating: ** (*** in minimum moves).

SOLUTIONS

Move definitions

1. *Two-dimensional sliding block puzzles.* A full discussion of all the possible variations is given in Chapter 1. Definitions used in this book are repeated below.

2. *Soko (warehouse) puzzles.* Definitions of a move are given at the beginning of Chapter 10.

3. *Railway shunting puzzles.* These definitions are given at the beginning of Chapter 11.

Move definitions of two-dimensional sliding block puzzles—Chapters 3–8.

Unless otherwise stated, only two definitions of a 'move' are used in this book:

(i) Slide *one* piece only in any direction or combination of directions. The piece may be slid any permissible distance without lifting or (unless specifically allowed) without rotating. (Under this definition a piece can move 'round a corner'; see Figure 11.)

(ii) Slide *one* piece only in any *one* direction. The piece may be slid any permissible distance without lifting or rotating (see Figure 12).

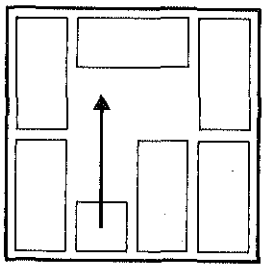


Figure 11

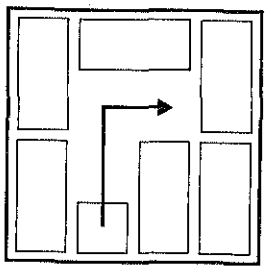


Figure 12

Definition (i) is the preferred definition. Where applicable, solutions are given for both, but so as to distinguish between the two, the number of moves for definition (ii) is always given in brackets. In the solutions a piece that changes direction (i.e. moves round a corner) is always underlined—see Notation below.

To count the total of moves under definition (i) it is only necessary to count the quantity of numbers. For definition (ii) totals, the number of underlines must be added to arrive at the correct figure.

Notation used in the solutions

Notation is given below for all the puzzles except where special notation applies to one particular puzzle alone. In this latter case it is given with the solution for that puzzle.

Numbers in the solutions refer to pieces, not positions. There are a few exceptions, which are clearly indicated at the beginning of the relevant solutions.

Where there is a choice of direction in moving a piece it is indicated by a letter after the piece number as follows:

^{U} = up
^{D} = down
^{L} = left
^{R} = right.

Sometimes there is a choice where pieces have to move first in one direction and then in another. In such cases the following letters are used:

^{UL} = up, left
^{UR} = up, right
^{DL} = down, left
^{DR} = down, right
^{LU} = left, up
^{LD} = left, down
^{RU} = right, up
RD = right, down

Pieces normally move as far as they can. Where a piece must move only half the possible distance it is marked with a $^{1/2}$ sign after the piece number. Where pieces have to make any other type of special move they have been given a special sign, which is then explained at the end of the solution in question.

Underlining indicates that a piece 'moves round a corner' [counting as one move under move definition (i) and two moves under definition (ii)]. If a piece has to be moved round two corners it is underlined twice [and counts as three moves under move definition (ii)]. In some half dozen instances it will be necessary to move a piece round three corners and this is indicated by a zigzag line under the piece number (e.g. 4).

A piece should always be moved as far as is allowed for a move unless otherwise stated. For example, it is often possible to move a piece round a corner, but *this must only be done if it is underlined.* If it is not underlined, it should be moved as far as possible without turning the corner. Similarly, when it is possible to move a piece round two corners, it must not be moved round any corners unless it is underlined and it must be moved round one corner only, and not two, if singly underlined.

Puzzles F5–F12 (Chapter 8) allow the rotation of pieces. Except in the case of F5, rotation is indicated by arrows over the pieces as follows:

$\overrightarrow{5}$ = rotate piece 5 clockwise 90°;

$\overrightarrow{7/8}$ = rotate pieces 7 and 8 together clockwise 90°;

$\overleftarrow{6}$ = rotate piece 6 anticlockwise 90°;

$\overleftrightarrow{4}$ = rotate piece 4 (in either direction) 180°.

Pieces being rotated may also be moved to a new position. This will be made obvious from subsequent moves.

The 'Soko' puzzles, H1–H10 (Chapter 10), require a different notation. As explained at the beginning of Chapter 10, the movement of the Sokoban (storeman) is free and does not count as a move. Therefore, only the movements of the other pieces (packing cases) are recorded. Each move is given in the form of a number in a circle followed by one or more letters and numbers.

(a) The first (number) in any sequence of numbers and letters is always the piece number.

(b) The second (letter) is the direction—L = left, R = right, U = up, D = down.

(c) The third (number) is the distance in units (squares) travelled.

(d) The fourth (letter) is as in (b).

(e) The fifth (number) is as in (c), etc.

For example, ② R^3 = move piece 2 to the right a distance of three units; ⑩ $L^1U^2R^4$ = move piece 10 to the left one unit, then up two units and finally right four units. (This last sequence of moves counts as one move under move definition (i) and three moves under definition (ii).

An asterisk at the end of a move sequence denotes that the piece has reached its final destination.

Parity changes (continued from Chapter 1)

As discussed in Chapter 1 the rules of parity show us that puzzles with uniform pieces can only be solved if the number of exchanges of pieces (to solve the puzzle) is even. If a parity check is taken on several of the puzzles in this book it will be found that the number of exhanges is odd and therefore, apparently the puzzle cannot be solved.

What the designers of these puzzles have done is to use a 'trick' in order to change a puzzle's parity and make it possible. The most common trick is to have two pieces that are identical. If these two pieces change places during the course of solving the puzzle an 'extra' exchange takes place. Parity is changed—from odd to even—and the puzzle becomes solvable. Other tricks are also used to change parity; rotating the whole puzzle 90° is one example; making a series of numbers read in a different direction (from that expected) is another.

Puzzles based on parity principles are: A1, A3, A7, A8, (B4), B7, B8, B9, (B10), B12, B18, B19, B20, B23, B24, B60, C2, C3, C48, C49, E12, E13, E22, E25, F1, (G2), G4.

Solutions

A1. When mixing up the pieces, slide the R from YOUR into the top left corner. The puzzle is impossible to solve with this R in this position.

A2–A6. No solutions needed.

A7. The solution is always possible because two letters (L) are the same. If the puzzle cannot be solved with one of the L's in one position, it will always be solvable in the other.

A8. The solution sheet gives elaborate directions on how to solve the puzzle for the three relevant end positions: (i) exchanging just two pieces on the outer circle; (ii) exchanging just two pieces on the inner circle; and (iii) both (i) and (ii) together. However, the puzzle becomes much easier when it is realized that there are two Ss on the outer circle and if the puzzle is solved so that this part of the puzzle is done last it becomes much easier.

A9. The problem is to find a pattern that meets all the conditions. Once it is realized that the four blue pieces can be pout in one basic configuration (see Figure 13) then the rest becomes quite easy. One each of the four remaining colours must occupy the four central squares and the rest is fairly obvious. The solver may find that, having decided on the pattern in Figure 13, he has realized that the four blue pieces can be put in one basic configuration (see

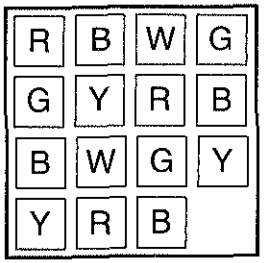


Figure 13

difficulty achieving it in sliding the pieces. The answer is to exchange two of the same coloured pieces and then the rest of the pieces should easily go into place. A rigid mathematical proof is given by Schwartz (1973), pp. 277–80. It states that there are only two solutions (excluding rotations and reflections).

A10. The problem is to find a pattern that fits the criteria. One example is given in Figure 14—there are many others.

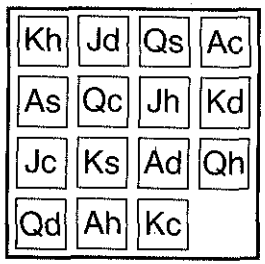


Figure 14

B1.
A B 3 A 1 2 A 3 B 1 3 A 2 3 1 B A. 17 moves.

B2.
S T P S T U E T U P S U P E T P. 16 moves.

B3.
2 6 3 4 6 1 5 6 4 3 1 4 6 5 4 1 3 6 5 4
1 2. 22 moves.

B4. Two possible solutions (Figures 15 and 16).

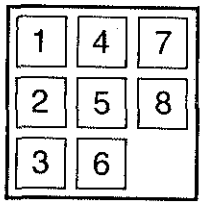


Figure 15

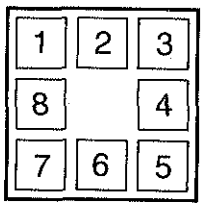


Figure 16

B5.
1 2 5 8 7 4 8 5 2 8 3 1 8 2 5 7 4 3 1 6
3 1 2 5 7 4 1 2 5 8. 30 moves.
There are nine other solutions in 30 moves.

B6.
A B F E C A B F E C A B D H G A B D H G
D E F. 23 moves.

B7.
I V A I R O T I I A V I I C I V A I.
18 moves.
One of six solutions in 18 moves.

An interesting solution in 22 moves is as follows:
A V I C T O R A V I C T O R A V I C T O
R I.

B8. This is a puzzle based on the parity principle. Both of the As would seem to start in their finish positions, but unless they exchange positions with each other, the puzzle is impossible.
G E L N A L E G L E C A N A E C A I G A
I N A E C L A I L A. 30 moves.

B9.
F L A F D E R D E N F A L S D E N F A N
S L N A F S L N A L. 30 moves.
No one said anything about having to read the word in a clockwise direction—this is impossible.

B10. The 14–15 puzzle is impossible, *unless* the 6 and the 9 are both turned upside down whilst solving it.

B11.
14 11 12 8 7 6 10 12 8 7 4 3 6 4 7 14 11 15 13 9
12 8 4 10 8 4 14 11 15 13 9 12 4 8 5 4 8 9 13 14
10 6 2 1. 44 moves.

B12.
14 15 10 6 7 11 15 10 13 9 5 1 2 3 4 8 12 15 10 13
9 5 1 2 3 4 8 12 15 14 13 9 5 1 2 3 4 8 12.
39 moves.

Now rotate the whole puzzle 90° clockwise!

B13.
15 14 13 9 5 1 2 3 4 8 12 15 14 13 9 5 1 2 3 4
8 12 15 14 13 9 5 1 2 3 4 8 12 15 14 13 9 5 1 2
3 4 8 12 15 14 13 9 5 1 2 3 4 8 12 15 14 13 9 5
1 2 3 4 10 11 7 6 11 10 8 11 10 7 6 10 11 12.
78 moves.

B14. The pattern to be reached is shown in Figure 17.

5	2	8	15
9	14	4	3
6	1	11	12
10	13	7	

Figure 17

12 11 7 3 2 1 5 9 10 7 14 15 7 6 1 2 4 8 3 14
15 7 13 10 6 1 14 15 11 3 15 4 8 15 3 12. 36 moves.
Note: There are two other patterns each having two solutions in 36 moves.

B15. The pattern to be reached is shown in Figure 18.

1	10	15	4
13	6	3	8
2	9	12	7
14	5		11

Figure 18

12 11 10 9 5 6 2 3 4 8 7 10 15 12 11 7 8 4 10 15
9 5 13 14 5 2 3 10 15 3 6 13 2 9 12. 35 moves.
Note: This seems to be the unique solution in 35 moves.

B16. The second part of each number refers to the position.
8–17 16–21 6–16 14–8 5–18 4–14 3–24
11–20 10–19 2–23 13–22 12–6 1–5 9–13. 14 moves.

B17.
9 6 7 4 3 7 6 8 5 2 7 6 8 5 2 7 6 8 5 9
4 3 8 1. 24 moves.

B18.
L A N A P C A P A N^D P A^D C A A P N A A C
P A C. 23 moves.

B19. R = red; W = white; B = blue.
G O R W B D O R O G W O R B O W G R B O^L
D O O^U B W O O^L D. 28 moves.

B20. R = red; W = white; B = blue.
G O R W O C U R W O B D C W O^R L R U W B
O O^U U W B C K. 27 moves.

B21.
8 4 3 CP CZ 3 4 7 CP CZ VT KW1 2 KW1 2 5 6 CP
CZ VT 3 4 7 8. 26 moves.

B22.
Y M R C L E F R E L C E M Y^D N A^R E C L F
R M Y N A X A E C L N C L A X A^D I A V X
A^D I E L C N A A N O E S M A S V I N X Y
S M A S^D Y S S A M S A Y V S A E L C N^R A
F R M Y O E S A S I S A E L C N A^U F R M
Y S A M Y V I S N X S^D I A N I S^R.
116 moves.

B23.

The solution to this puzzle is given in terms of the movement of the pieces, as follows:

U = up; D = down; R = right; L = left.
R D D L L U R R D L U R U L D L D.
17 moves.

B24. Number the pieces as follows: top row left to right 1, 2, 3; second row 4, 5, 6; third row 7, 8; bottom row 9, 10, 11.
5 2 1 4 2 6 3 1 6 5 8 3 1 6 5 8 3 11 10 3
11 1 8 2 7 9 3 10 1 11 10. 31 moves.
Now rotate the whole puzzle 180°!

B25.
2 9 K 3 4 5 6 7 8 A Q K 3 4 5 6 7 8 A Q
K 3 4 5 6 7 8 A Q K 3 4 5 6 7 8 A Q K 3
4 5 6 7 8 A Q. 47 moves.
Rotate the whole puzzle 180°! In rotating the puzzle, the 6 on the base of the tray will change to 9, as in the diagram.
The diamond suit reads the same either way up, the only exception being the 7. In the diagram showing the start position the artist has cheated by putting the central diamond of the seven in the centre of the card. It should be either in the centre of the top half or bottom half.

B26. The solution to this puzzle is given in terms of the movement of the pieces, as follows:
U = up; D = down; R = right; L = left.
D R R U R R D L D D D L U U U U L D D D
D R U U R D D L U U U U R D D D L U R D
L L D R R U U L D L U R U U R D L L D R
R U R U L L L D R U L D L U. 74 moves.

B27. The solution to this puzzle is given in terms of the movement of the pieces, as follows:
U = up; D = down; R = right; L = left.
Remove piece at top right corner.
U R U R R D D L U U R D L D L U L U
18 moves.

B28. The solution to this puzzle is given in terms of the movement of the pieces, as follows:
U = up; D = down; R = right; L = left.
Remove piece upper right of centre.
R U L L D R R R U L L D R D L L U U R U
R D D D R. 25 moves.

B29.
11 8 7 4 5 6 9 11 8 10 4 7 11 8 10. 15 moves.
The sixth magician does not disappear—all the others get slightly larger.

B30.
11 8 7 4 5 7 9 6 3 2 1 5 7 3 6 11 8 10 4 7
5 1 3 9 11 8 10. 27 moves.

B31.
G^{L} R B Y Y B R R B R R G R B Y R G Y B R.
20 moves.

B32.
Y G G^{L} Y R Y G G Y B B Y G^{D} R B G R B Y G.
20 moves.

B33. In the next seven puzzles, numbers refer to pieces, letters to positions.
2-E 5–B 4–D 2–H 4–I 5–G 3–E 5–C 4–B 6-D
3–J 6–I 4–G 1–E 4–A 6–B 1–I. 17 moves.

B34.
2–E 5–B 4–D 2–H 4–E 5–I 1–G 4–A 1–E 5–B
2–D 1–H 6–E 2–J 5–I 3–G 6–C 3–E 5–B 2–D
3–J 2–I. 22 moves.

B35.
6–G 2–B 1–E 3–H 4–I 3–L 6–K 4–G 1–I 2–J
5–H 4–A 7–F 8–E 4–D 8–C 7–A 8–G 5–C 2–B
1–E 8–I 1–G 2–J 7–H 1–A 7–G 2–B 6–E 3–H
8–L 3–I 7–K 3–G 6–I 2–J 5–H 3–C 5–G 2–B
6–E 5–I 6–J. 43 moves.

B36.
3–J 2–I 1–G 3–A 8–B 7–C 6–F 1–K 2–L 6–G
7–I 8–M 3–J 6–A 7–B 4–G 5–I 3–E 8–D 9–C
10–F 5–O 4–N 10–I 9–G 8–M 3–J 10–E 9–D 3–G
8–C 3–M. 32 moves.

B37.
2–1 3–2 4–3 5–11 6-4 7–5 8–6 9–7 1–13 9–10
8–9 1–12 7–13 6–8 5–7 1–11 4–12 3–6 2–5 1–1
2–2 3–3 4–4 5–5 6–6 7–7 8–8 9–9. 28 moves.

B38. Number the pieces as in the previous puzzle (B37).
2–1 3–2 4–3 8–13 7–8 6–7 5–12 6–11 7–4 8–5
5–13 9–6 1–7 5–10 1–13 9–9 8–8 7–12 4–7 3–6
2–5 6–1 2–2 3–3 7–11 4–12 8–4 1–5 4–13 1–12
9–7 4–9 9–13 8–8 3–7 1–3 7–12 1–11 2–6 6–5
1–1 6–2 2–11 7–3 3–12 8–4 9–5 3–13 9–8 8–7
2–12 8–4 9–5 2–6 3–7 4–8 5–9. 57 moves.

B39.
4–H 6–F 8–G 10–I 2–N 10–J 8–M 4–L 10–H 6–J
3–K 1–I 10–E 5–H 7–G 9–F 10–A 9–E 7–B 5–C
9–H 1–E 3–D 6–F 9–J 4–H 8–G 2–I 9–N 2–M
8–J 4–L 8–H 6–J 3–K 1–J 8–E 5–G 7–F 8–B
7–E 5–C 7–H 1–E 3–D 6–F 7–J 4–G 2–I 7–M
2–L 4–J 6–H 3–K 5–I 6–C 5–H 3–D 2–F 5–L
2–H 3–K 1–I 2–E 1–H 3–D 4–F 1–J 4–H 3–K
4–D. 71 moves.

B40. Numbers refer to rows. In any start position Y must be the last letter for the puzzle to be possible.
Y4 L4 E4 C4 I4 U4 M4 N1 T2 M3
T4 N2 O1 O1 R1 N1 A1 C1 H1 E2
L2 P2 S2 P2 N2 O3 N3 T3 U3 P4
O4 R3 S1 P1 O1 R2 P3 R4 L4 E2
H2 C2 A2 N2 S2 P1 O1 P1 U1 L1
R1 I3 E4 I4 H3 C4 R3 A1 R1 N3
C2 I2 E2 N2 C2 E2 L3 Y3. 68 moves.

B41. Number the nine positions as follows: top row, left to right—1, 2, 3; middle row, left to right—4, 5, 6; bottom row, left to right—7, 8, 9. Letters refer to pieces, numbers to positions.
A–3 D–7 E–1 A–9 D–3 A–5 C–7 A–6 E–9 B–5
D–4 B–1 C–3 B–7 E–5 D–1 A–9. 17 moves.

B42. Pieces move as far as permitted unless otherwise indicated.
3 2 1^{R} 10 9 8 4^{DL} A 4 8 9 10 7 A^{DLD} 1 2 3 A.
18 moves.

B43. In each move a piece can only be moved to an immediately adjacent square.
M L^{D} A I N E^{L} Y^{D} D I N L D I N L^{U} E N L A^{R} M
M^{U} E N L I D. 26 moves.

B44.
T I H W H I T C A L B W I H T H I B L A
C H I T B L A C H I T $L^{½}$ A C H I T^{U} E L A
C H I T E A C L C A K E T I H L A C K C
A L. 62 moves (85).

B45. No solution given.

B46.

1 2 3 4 1 7 6 5 1 7 2 3 4 7 2 6 5 1 2 6
3 4 7 6 3 5 1 2 3 5 4 7 6 5 4.

35 moves.

B47.

1 2 3 4 1 7 6 5 1 4 3 2 7 6 5 1 4 6 5 1
4 6 3 2 7 5 1 4 6 3 1 5 7 2 1 5 4 6 3 5
1 2 7 4 1 5 3 6 1 4 7 2 5 4.

54 moves.

B48.

1 6 3 4 1 7 2 5 1 7 6 3 4 7 6 2 5 1 6 2
3 4 7 2 3 5 1 6 3 5 4 7 2 5 4.

35 moves.

B49.

1 6 3 4 1 7 2 5 1 4 3 6 7 2 5 1 4 2 5 1
4 2 3 6 7 5 1 4 2 3 1 5 7 6 1 5 4 2 3 5
1 6 7 4 1 5 3 2 1 4 7 6 5 4.

54 moves.

B50.

4 5 1 2 3 6 9 C 12 11 10 7 5 1 2 3 6 9 C 12
11 10 7 5 1 2 3 4 2 1 5 7 10 11 12 C 9 6 4 3
1 5 7 10 11 12 C 9 6 4 3 2 5 7 10 11 12 C 9 6
4 3 2 1 7 10 11 12 C 9 6 4 3 2 1 5 10 11 12 C
9 6 4 8 6 4 8 3 2 1 5 10 11 7 10 11 7 12 C 9
4 8 3 6 8 3 6 2 1 5 11 7 12 10 7 12 10 C 9 4
3 6 2 8 6 2 8 1 5 11 12 10 C 7 10 C 7 9 4 3
2 8 1 6 8 1 6 5 11 12 C 10 9 4 3 2 1 6 5 8
6 1 2 3 4 7 10 C 12 11 8 5 1 2 3 4 7 10.

178 moves.

C = clockface.

B51–B59. No solutions given.

B60. At first sight this puzzle seems impossible. To solve the puzzle it is necessary to exchange the 1 with the 15 and this would appear to go against the laws of parity, which say that it is impossible to exchange any two pieces (see Chapter 1). However, the puzzle is so constructed that its parity can be changed, if one knows the secret.

The puzzle consists of three circuits each having six numbers (note that it is an *even* number) plus the centre spot: 1–6, 6–11, 11–1. If it is possible to move just one piece (or any *odd* number of pieces) around one circuit without moving any other pieces around any circuit, then parity can be changed and the puzzle solved.

In searching for a solution with the fewest moves possible, the shortest that moved just one piece around a circuit had 89 moves. The shortest solution of all, believed to be the minimum, moves seven pieces around the top left circuit and has 81 moves.

The solution given is not unique—there are at least two others having 81 moves.

14 13 12 11 15 6 7 8 9 10 15 6 1 14 13 12 11 6 1 14
13 12 11 6 1 15 10 9 8 7 5 4 3 2 13 12 11 6 1 15
14 12 13 2 3 4 5 12 13 11 6 1 15 14 13 11 6 1 15 14
13 11 12 7 8 9 10 11 12 6 1 15 14 13 12 11 10 9 8 7
6.

81 moves.

C1.

1 B 4 5 3 2 B 1 A 4 5 3 2 B 3 4 5 A 1 3
B 2 4 5 3 1 A 3 4 5 2 B 1 A 3 4 5 1 B 2
1 4 5 3 A B 2 1 4 5 B A 3 B A 3.

56 moves.

C2. At first sight the puzzle looks impossible; but if the puzzle is looked at closely it will be seen that pieces 2 and 6 are identical and if their positions are exchanged, the puzzle is then very easy. The point of the rectangular piece is to prevent having two identical (blank) pieces, which would make the puzzle similar to B23.

Early solutions had 36 or even 42 moves but this has recently been reduced to 28, which is believed to be the minimum.

5 6 7 5 6 4 3 2 9 6 2 9 6 G 1 6 9 3 4 2
5 7 2 5 G 9 6 1.

28 moves.

C3. Move definition as in the text.

T H I N K H * T H I N K H + ? D R A * T
H I N K H + ? D R A * T H + ? D R A * ?
+ K H H T ? * A R D + K H N I H T ? * A
R D + K H N I H T ? * A R D + K H N I H
T ? * H K + (35).

35 moves (__ = one move). If singly, 87 moves.

Note that the two 'H' pieces have to exchange places in order for it to be possible to solve the puzzle.

C4.

9 8 2 4 7 5 1^{D} 3 6 8 2 5 1^{D} 3 6 2 9 8^{D} 2 9
8 5 8 3 1^{R} 7 1 8 5 2.

30 moves (33).

C5.

Ⓛ E R D O Y M R D O Y M R D E Ⓙ D E O Y
M W W E T D E R E W W E W I Ⓜ W I Ⓜ R T
Ⓜ W R Ⓜ W R^{R} T W D E Ⓜ D W H E W H T R I
W R I $H^{2/3}$ W E T I H W R H $I^{2/3}$ T E W W I T E
W W $I^{2/3}$ R H T E W W. 89 moves.

Ⓜ = man; Ⓛ = lady; $^{2/3}$ = move only two-thirds way.

C6.

7 8 4 6 7 5 3 9 10 7 5 6 2. 13 moves (17).

C7.

4 5 2^{D} 10 4 10 5 2^{L} 3 10 7 6 5 1 8 4 5 1 2 3.
20 moves (23).

C8. Move definition (i).

7 4 3 4 6 10 7 4 6 10 4 6 10 3 10 6 7 2 4.
19 moves.

C8. Move definition (ii).

7 4 8 5 3^{D} 10 6 7 4 10 3 5 8 10 6 7 4^{R} 6 7 2
4. (22) moves.

C9.

A 1 15 4 5 6 7 11 10 14 A 13 12 10 11 9 4 15 1 13
12 10 11 9 4 15 A 14 9 4 15 A 3 6 8 5 A 15 4 11
10 12 13 1 2 6 A 5 8 A 6 2 1 13 12 10 11 4 7 A.
60 moves.

C10.

A 15 14 A 1 2 3 6 7 8 11 12 13 A 14 15 1 4 A 13
12 11 8 7 6 3 2 5 A 13 14 15 12 10 9 A 5 2 3 6
7 8 9 13 4 2 3 6 7 A 13 9 8 A. 54 moves.

C11.

A 1 3 4 7 8 9 10 12 15 A 14 13 12 15 A 2 5 7 8
9 10 11 A 15 12 13 14 2 5 7 A 11 10 9 6 A 8 6 9
10 11 8 6 9 A 6 8 11 10 A. 51 moves.

C12.

4 5 6 7 2 3 5 6 8 7 2 3 6^{R} 5 1 4 A 4 1 5
6 A^{DR} 6^{R} 3 2^{L} 7 9 A. 28 moves (35).

C13.

2 3 4 5 6^{D} 4 5 6 4 A 2 1 3 5 4 A 2 1 3 5
4 6 A. 23 moves (27).

C14.

10^{L} 4 $8^{1/2}$ 6 1 2 9 10^{D} 4 8 6^{U} 2 10 7 4 9 5 3^{U} 4 9
8 6 2 8 7 9^{R} 4 3 6 2 1 5 6 2 3 4 9 7 8 1
5 6^{D} 2 4 7 9 3^{D} 4 6 7 9 3 $4^{1/2}$ 6 9 3 8 10 1 5
7^{R} 3 $9^{1/2}$ 10 2 1 7 5^{D} 3 10. 70 moves (76).

C15.

5 3 4^{D} 2 1 3 5 4 2 1 3 5^{U} 4 2 1 4 9 6 7 8
6 7 9 5 3 4^{U} 5 2 8 7 9 A 3 4 2 5 8 1 7 6
9 A 4 2 8 A 4 3 2 8 5 1 A 4 9 6 7 A 4 3
9 7 A. 63 moves (70).

C16.

7 9 8 7 9^{L} 8 3 1 2 4 5 1 2 A 7 3 8 9 6 1
2 8 A 4 5 8 1 2 9 A 4 7 3 A 4 1 2 9 6 A.
40 moves (45).

C17.

4 5 3 1 A 4 5 3 1 2 A 5 3 B 7 8 6 1 2^{D} A
5 4 3 B 7 8^{U} 6 1 2^{D} A 4 5^{D} 3 B 8 A 2 1 6 7
8 A 1 6 7 8 A. 47 moves (55).

C18.

8 7 A 1 2 7 8^{U} 3 4 5 6 A 7 8 2 $1^{1/2}$ 7 8 2 3
4 A 8 7 1 2 3 4 A 8 6 5 A. 33 moves (34).

C19.

8 7 A 1 2 7 A 6 5 4 3 8 7 A 6 5 3 7 8 4^{R}
3 7 8 4 3 7 5 6 8 7 5 $6^{1/2}$ 8 7 5^{U} 3 4 A 7 8
1 $2^{1/2}$ 7 8 1 6 5 A 8 7 1 2 6 5 A 8 4 3 A.
59 moves (62).

In the puzzle with words on the pieces, five extra moves are required: 5 6 2 1 7. The sentence reads: 'Dear Sir, You can now (under)stand and (over)awe those (under)me.' Words in parentheses are implied.

C20.

8 7 A 1 2 8 3 4 7 8 A 6 5 7 8^{L} 4 3 A 8 7
5 6 7 8^{L} A 3 4 5 6 7 8 A 5 6 8 A 1 2 3 4
5 6 8 7 A. 45 moves (47).

C21.

8 7 A 1 2 8 3 4 7 8 A 6 5 7 8^{L} 4 3 A 6 $5^{1/2}$
7 8 6 1 2 A. 26 moves (27).

C22.

7 8^{U} 3 4 6 10 3 8 7^{R} 3 10 1 A 4 5 8 9^{L} 2 4 5^{R}
A 1 6 8 A 4 5 2 9 11 7^{R} 10 A 5 2 9 11 A 8 6^{D}
3 10 7 A 2 9 11^{D} 2 8 6 3 10 7^{L} A 2 6 3 1 4 5
8^{R} 3 6 8 2 A 7 10^{R} 1 6 10 7 A 2 11 9 5.

77 moves (90).

C23.

8 6 4 2 A 5 7 8 6 10 4 2 A 5 7 8 A 2 4 6
A 8 1 3 9 10 6 A.

28 moves (34).

C24.

10 8 6 5 2 1 A 4 5 2 1 A 4 3 5 1 2^{U} 6 7 A
1 2 6 7 8 10 9 A 7 8 9 A.

32 moves (35).

C25.

10 8 7 3 1 A 5 4 3 1 9 10 8 7 3 1 9 A 5 4
1 3 7 8 9 10^{D} A 4 1 3 6 7 8 9 10 A 7 8 10 A.

40 moves (51).

C26.

7 8 4 3 6 7 8 10 4 3 7 6 5 1 A 6 5 8 1 9
10 4 3 8 1 9 A 6 5 1 8 3 4 9 10^{D} A 5 1 8 7
3 4 9 10 A 3 4 10 A.

49 moves (64).

C27 (a). Move definition (i).

3 7 6 4 3 7 6^{R} 4 1 A 8 6 7 5 9 2 8 6^{R} A 1
4 7 A 8 6 2 9 5 7 A 8 1 4 A 9 5 7^{R} 9 A 4
1 6 8^{L} 2 5 A 8 6 1 4 8 3 9 7 A 6 8 3 7 A.

60 moves.

Move definition (ii).

3 8 7^{D} 6 4 3 8 7^{R} 4 1 A 6 7 8 5 9 2 6 7^{R} A
1 4 8 A 6 7 2 9 5 8 A 6 1 4 A 9 5 8^{R} 9 A
4 1 6 7^{L} 2 5 A 6 7 1 4 6 3 9 8 A 7 6 3 8
A.

(67) moves.

C27 (b).

3 7 5 2 A 1 4 8 3 5 6 8^{R} 4 1 A 2 6 8 7 4
1 A 7 8 6 2 7 8^{R} A 1 4 6 A 7 8 2 9 5 6 A
7 1 4 A 9 5 6^{R} 9 A 4 1 7 8^{L} 2 5 A 7 8 1 4
7 3 9 6 A 8 7 3 6 A.

70 moves (81).

C27(c).

6 8 4 6 8 3 7 9 4 8 3 9 7 4 5 8 6 3 7 4
5 2 A 1 9 7 4 1 A 2 5 6 8 3 1 4 6 A 9 7
6 4 1 8 A 9 2 5 A 9 $7^{1/2}$ 6 4 1 8 9^{D} A 5 2 6
7^{R} 4 1 A 7 6 2 5 7 3 9 8 A 6 7 3 8 A.

78 moves (89).

C27(d).

$9^{1/2}$ 5 3 6^{D} 4 $8^{1/2}$ 6 4 3 7 9 4 8 3 9 7 4 5 8 6
3 7 4 5 2 A 1 9 7 4 1 A 2 5 6 8 3 1 4 6
A 9 7 6 4 1 8 A 9 2 5 A 9 $7^{1/2}$ 6 4 1 8 9^{D} A
5 2 6 7^{R} 4 1 A 7 6 2 5 7 3 9 8 A 6 7 3 8
A.

81 moves (90).

C28.

D^{L} M R B U P L C B R M D P U L C B R L P
U M L R G W L $R^{1/2}$ B C P U M^{L} R L W G B C^{R} P
U L B C G W B D R M L C B D M L $C^{1/2}$ B D M
R.

61 moves (70).

C29.

A^{R} 5 7 3 5 7 A 6 8 3 7 A 8 6 4 3 7 5 A 6
3 5 7 A 6 8 3 7 5^{R} 4 2 B 5 7 6 8 A 4 2 B
5 7 6 B 2 4 8 B 7 5 2 4 8 A.

54 moves (64).

C30.

7 5 3 2 1 4 6 7^{L} A 1 4 6 7 A $1^{1/2}$ 3 5 9 8 A
7 6 4 2 3 1 4 6 2 1 3 4 5 6 7 A 8 9 6 7^{D}
4 5 3 1 2 A 7 6 9 8 6 7^{D} A 2 1 3 4 5 9 8
6 7 A 9 8 7 A.

67 moves (71).

C31.

5 2 1 3 4 A 7 6 5 A 4 3 1 9 8 7 4^{D} A 5 6
4 A 3 1 9 8 7 4 A 8 1 9 2 3 8 5 6 A 7 1
9 2 8 3 5 6 A 7 4 1 9 2 3 5 6.

55 moves (64).

C32.

7 8 6 5 4 3 1 A 9 8 4 3 1 2 A 7 8 9 7^{R} A
2 1 8 A 7.

25 moves (30).

C33.

2 6 7 8 9 5 4 A 1 2 3 8 9 4 A 1 3 2 6 8.

20 moves (24).

C34. Move definition (i).

9 8 6 7 3 9 8 A 1 2 4 5 3 $8^{1/2}$ 5 3 6 7 A 1
2 9 5 8 3 $4^{1/2}$ 5 8 3 6 7 A 1 9 8 5 3 4 1.

39 moves.

Move definition (ii).

9 8 6 7 3 9 8 A 1 2 4 5 3 $8^{1/2}$ 5 3 6 7 A 1
2 9 5 8 3 $4^{1/2}$ 5 8 3 6 7 A 1 2 9^{U} 8 5 3 4 1.

(44) moves.

C35. Move definition (i).

9 8 7 6 5 4 3 1 A 9 8 6 7 5 4 3 6 A 2 1
6 A 7 8 9½ 2 7 8 3 A 1 7 2 8 9 3 5 4 A 3
8 9 2 7 1 3 A 4 5 8 7 2 9^{U} 8 7 2 9 8½ 7 2
9 1 6 3 A 4 5 2 7^{D} 4 A 3 6 1 8 9 4 2 7^{U} 5
A 4 7 8 9 1 6 3 4 A 5 2 7 1 6 9 8 2 7 5
A.

101 moves.

Move definition (ii).

9 8 7 6 5 4 3 1 A 9 8 6 7 5 4 3 6 A 2 1
6 A 7 8 9½ 2 7 8 3 A 1 6^{D} 7 2 8 9 3 5 4 A
3 8 9 2 7 6 1^{U} 3 A 4 5 8 7 2 9^{U} 8 7 2 9 8½
7 2 9 1 6 3 A 4 5 2 7^{D} 4 A 3 6 1 8 9 4 2
7^{U} 5 A 4 7 8 9 1 6 3 4 A 5 2 7 1 6 9 8 2
7 5 A.

(110) moves.

C36. Move definition (i).

8 9 7 6 5 3 1 A 9 8 1 3 4 2 A 9 8 1 5 4
3 1 5 7 6 4 2 1.

28 moves.

Move definition (ii).

8 9 7 6 5 3 1 A 9 8 1 3 4 2 A 9 8 1 5 4
2^{D} 3 1 5 7 6 4 1 2 3^{D}.

(34) moves.

C37.

3^{R} 8 9 A 2 3 8 9 A 2 1 3 8 4 5 9 8 4 5½ 9
8 4 3 1 2 A 6 7 8 9 4 5 3 2 A 7 8½ 9 6 3½

40 moves (44).

C38.

9 8 A 1 2 8 A 7 6 5 4 3 9 8 A 7 1 2 A 7
6.

21 moves (23).

C39.

4 6 5 3 4 9 8½ 7 9 4 6 5 9 8 4 5 6 3 9 8
7 4 5 6^{U} A 1 2 3 9 7 8^{D} 4 6 5^{U} A 1 2 3 9 7
8^{D} 4 6 5 A 7 8 4 5 A 7 8 4½.

53 moves (57).

C40.

A 9 1 2 3^{L} 9 A 6 4 5^{R} 9 2 3^{R} 1 A 9 4 5^{L} 6 9
A.

21 moves (21).

C41.

9 5 4 8 9^{U} 5 4 9 3 6 8 1 A 6 8 3 9 7 2 6
8^{R} A 1 9 5 4 2 7 3 9 5 1 A 8 3 2 9 7 4 1
A 8 6 3 9 A 8 6 3 9 2 7 4 5 1 8 6 A 7 9
2 3 A 6 8 1 4 5 2 9 7^{U} 1 6 A 3 7 9^{U} 1 2 5
4 6 8^{D} A 3 7 9 1 2 5 4 6 8 A 5 4 8 A.

98 moves (108).

C42(a).

5^{R} 6 8 9 7 4 2 A 1 3 6^{L} 1 A 2 4 7 9 8 1 5
7 9 8 1 6 5 1 9 8 4 7 4 8 6 5 1 9 6 8 4
9 1 5 8 6 1^{R} 5 3 A 7 9 1 6 4 8 5 3 A 7 2
9 4 6 1^{D} A 3 5 8 6 1^{R} A 2 7 3 5 8 6 1 9 4^{U}
A 5 3½ 9 2 7 5 8 6 1 A 7 2 4 9 5 3 8 1 A.

100 moves (113).

C42(b). The first hundred moves are the same as C42(a). Then

4 7 8 1 3 5 9 7 2^{L} 8 1½ 2 8 4 A 2 3 6 2 3
6^{R} 5 8 9 7 9 8 6 3 A 4 7 9 8 6 5 2 A 1 7
4 1^{R} A 2 5 6 8 9 7 3½.

150 moves (171).

C43.

4 5 7 4 3 8 9^{L} 4 7 5 3 4 6 9 8 4 3 7 6 5
7 6 A 1 2 3 8 9 4 5 6 A 3 9 4 5 6 7 A 8
6 7 5 4 9 3 6 7 9 3 2 1 6 8 9.

55 moves (68).

C44.

4 5 7 4 3 8 9^{L} 4 7 5 3 4 6 9 8 4 3 7 6 4
8 3 6 7 A 1 2 7 A 5 4 8 9 3 6 7 A 5 4 8
9 3 7 A 2 1 4 5 3 9 7 6 A 3 7 5 4 7 9 8
6 5 4 7 9 8 6^{D} 5 4 6.

70 moves (80).

C45.

4 5 7 4 3 8 9^{L} 4 7 5 3 4 6 9 8 4 3 7 6 4
8 3 6 7 A 1 2 7 A 5 4 8 9 3 6 7 A 8 6 7
A 2 1 8 6 5 4 9 3 A 2 1 6 8 4 5 2 A 3 7
9^{U} 2 4 5 8 6 1 A 2 7 9^{D} 3 2 A 1 6 8 4 5 7
9^{D} 3 2 A 1 8 5 4 9 7 1 8 6 5 8 6 1 7 9 4
8 4 1 5 6 4 9 7 1 4 6 4 5.

113 moves (127).

C46.

6 7 9 5 4 9 7 6 8 10 5 4 9 7 6 8 10 5 4 7
6 8 A 1 2 6 7 4 5 8 10^{R} A 2 6 3 9 7 4 5 8
10 A 6 1 2^{L} 6 1 3 7 4 5 10 A 6 1 3 2^{D} 1 3 4
5 10 8 A 6 3 4 10 8 5^{D} 4.

71 moves (82).

C47. Move definition (i).

6 5 8 9 10 7 3 1 2 A 4^{U} 5 9 10 7 A 1 2 A 3
7 5 9 7 6 8 10^{R} 7 9 4 5 1 2 A 9 6 3^{D} A 1 2
4 5 9 6 3 8 10 7 5 6 3 8 A 9 2 1 3 8 10 A
2 1 3 4 6 5 6 9 7 A 10 8 9 6^{D} 3 4 1 2 8 10^{U}
A 7 6 4 3 9 A 10 8 9^{R} 1 3 A 8 9 1 3 5 4 A
10 7 6 A 10 5 3 2 1 8 3 5 10 A 6 7 9* 8 3 5
A 4 10^{L} A 9 8 5 3 1 2^{R} A 9 4 10^{D} A.

* = move only half way on second part of move.

135 moves.

Move definition (ii).

6 5 8 9 10 7 3 1 2 A 4^{U} 5 8 9^{D}10 7 A 1 2 A
3 7 5 8 7 6 9 10^{R} 7 8 4 5 1 2 A 8 6 3^{D} A 1
2 4 5 8 6 3 9 10 7 5 6 3 9 A 8 2 1 3 9 10
A 2 1 3 4 6 5 6 8 7 A 10 9 8 6^{D}3 4 1 2 9
10^{U}A 7 6 4 3 8 A 10 9 8^{R} 1 3 A 9 8 1 3 5 4
A 10 7 6 A 10 5 3 2 1 9 3 5 10 A 6 7 8* 9 3
5 A 4 10^{L} A 8 9 5 3 1 2^{R} A 8 4 10^{D}A.

(158) moves.

* = move only half way on second part of move.

C48, C49. These puzzles are based on the parity principle: to move either the N or the W to the bottom right corner and leave all the other letters in their same positions is impossible unless two identical pieces exchange places. This is not so easy as it looks, as the rectangular blocks can only move lengthwise and there is not a lot of space. To exchange two similar pieces—the choice is only the Is or Ts—requires a minimum area of 3 × 2. This space, which has to be created, must be occupied by five small squares, of which two must be the Is or Ts. In C48 the Ts change places in moves 27–35, and in C49 the Is change places in moves 17–31.

C48.

D N A H F W I A P E N D E V O L H F P E
N D A F P E I T^{D} H H D N I T T D H H T T
I D T I E P F A N D E P F H L O V E N D
E P A I W F H A.

68 moves.

C49.

D N A H F W I T I P E N A A T I^{R} W T I I
P I I W T I I P W I A A N H T P W I T I
F H A N H T P W I T A E W P T H D E N W
H D W N E W D H E A T I P T H E P I T I.

80 moves.

C50.

3 10 9 3 4 10 9 A 1 2 9 10 4 3 A 9 10 3 A 8
7 5 6 4 3 A 8 7 1 6 3 A 10 9 7 8 5 6 3 4
A.

41 moves (44).

C51.

3 10 9 3 4 10 9 A 1 2 9 10 4 3 A 9 10 3 A 8
7 5 6 4 3 A 8 7 5 3 4 6^{R} 5 3 4 A 10 9 8 4
3 7^{D} 8 4 3 7 8 4 9 10 3 4 9 $10^{½}$ 3 4 9 8 7 A
4 3 9 10 8 7 A 4 6 5 A.

71 moves (75).

C52.

1 5 6 2 1 5 6 1 3 4 1 2 3 $4^{½}$ 1 2 3 5 6 A
1 2 3 4 5 6 A 2 4 3 7 8 10 9 5 6 A 2 1 4
3 7 8 10 9 5 6 A 1 3 4 7 8 A 1 2 3 4 7 8
A 1 5 6 9 10 A 7 8 4 3 2 6 5 A 8 7 1 2^{L} 6
5 A 8 $7^{½}$ 2 1 8 10 9 A.

90 moves (96).

C53.

9 8 7 5 4 3^{L} A 1 2 6 A 3 4 5 7 6 A 4 5 7
6 A 2 $1^{½}$ 3 4 2 9 8 A 6 7 5 10 11 12 6 7 5 2
4 3 10 11 12 7 13 A 2 5 13 6 7^{R} 14 A.

55 moves (63).

C54.

1 2 A 3 4 5 A 1 2 7 8 6 5 A 1 2 7 8 5 A
10 11 12 13 6 5 A 10 11 12 13 5 A 7 8 2 1 10 11 12
13 5 6 A.

44 moves (47).

C55.

5 A 1 2 5 6 3 4 2 5 6 12 14 9 3 14 12 6 A 7
8 11 A 6 14 3 9 12 A 11 13 12 A 11 6 14 11 3 9 A
13 6 11 3 9 4 10 A.

48 moves (55).

C56.

8 9 10 11 13 9 8 5 6 4 3 1 2 A 12 13 9 8 5 6
4 3 1 2 A 13 8 9 6 5 A 13 12 8 9 6 5 A 13 1
2 3 4 A 13 12 1 2 3 4 A 13 10 7 A.

55 moves (59).

C57.

8 9 10 11 13 9 8 5 6 4 3 1 2 A 12 13 9 8 5 6
4 3 2 1 A 13 8 9 11 10 6 5 A 13 12 8 9 11 5 A
12 13^{D} 1 2 13 12 1 2 13 3 4 12 13 3 4 12 13^{R} A 5 11
9 8 1 2 3 4 13 12 A 5 11 9 8 1 2 3 4 12 A 7
6 10 8 9 11 3 4 2 1 11 8 9 10 6 7 A 12 13^{D} 2 3
4 5 6 10 8 9 5 3 4 1 11 5 8 9 10 7 A 12 13^{D} 2
1 3 4 6 7 10 8 9 6.

129 moves (133).

C58.

11 12 4 5 6 8 7 4 12 11 A 1 2 11 A 10 9 4 7 12
11 A 10 9 4 11 12 7^{R} 4 11 12 A 2 1 10 12 11 9^{D} 10 1
2 3 5 6 A 11 12^{R}9 10 1 2 3 5 6 A 11 7 8 A.

59 moves (61).

C59.

8 2 1 8 7 2 1½ 8 7 2 5 3 A 7 8 1 2 4 11 12
10 9 6 3 A 8 2 1 13 12 11 4 5 A 8 7 2 1 13 12
11 4 5 A 8 3 6 A 4 5 9 A 3 6 7 8 4 5 9 10
A.

61 moves (67).

C60.

11 10 9 A 1 2 9 10 11 3 4 2 9 10 A 8 7 5 6 B
4 3 11 B 6 5 A 10 11 B 6 5 A 10 8 7 A.

37 moves (40).

C61.

11 10 9 7 8 6 5 9 10 11 B 5 9 11 B 4 3 1 2 10
11 B 4 3 1 2 10 11^{R} A 8 7 B 11 10 A 8 7 B 11 9
6 B.

42 moves (44).

C62.

11 12 A 4 13 12 A 3 2 10 9 11 14 A 3 2 10 1 11 14^{U}
A 13 5 6 7 8 B 14 11 A 13 12 5 6 8 B 11 A 12 5
6 8 7 B 11 14 A.

47 moves (58).

C63.

11 12 3 2 1 9 11 8 7 6 5 4 13 A 3 2 12 A 13 14
7 A 12 10 8 11 B 6 5 A 11 9 B 6 5 A 9 B 1 8
10 B 9 A 5 6 9 A 11 12 B 10 8 1 9 A 11 5 6 A
11 12 7 14 13 B 8 2 3 B 13 4 5 6 7 8 12 11 9 1
2 3 12 11.

84 moves (102).

C64.

15 14^{U} 9 6 1 A 4 5 1 11 10^{D} 9 6 8 1 10 A 2 3 12
13 9 12 A 10 11 12 A 10 2 3 13 9 A 12 11^{U} 1 8 6 14
A 11 12 6 14 15 A.

47 moves (54).

C65.

15 14^{U} 9 6 1 2 3 12 13 14 15 9 6 1 10 3 2 12 13 14
15 11 10 3 2 12 A 4 5 2 3 12 13 A 4 5 2 3 13 11
15 14 A 11 15 14 A 11 15 14 13 1 6 9 10 12 A 13 14 1
6 9 12 A 14 15 1 6 9 12 10 A.

72 moves (85).

C66.

14 9 6 11 13 15 14 9 6 8 5 11 13^{D} 1 2 3 12 14 9 6
8 5 7 4 11 A 3 2 10 15 8 5 7 4 11 13 A 10 15 14
12 9 6 5 7 4 13 A 10 15^{L} 3 2 14 1 4 13 11 A 15 4
13 11 A 15 10 4 11 13^{D} 8 7 A 11 13 8 7 6 5 A.

78 moves (89).

C67.

5 4 3 2 1 14 13 A 2 1 13 A 11 12 9 10 7 6 2 1
13 14 $A^{\dagger}$ 11 12 9 10 7 1 2 6 7 8 5 6 3 4 14 13 A
12 11 2 7 6 3 4 13 A 2 6 3 4 13 14 $A^{\dagger}$ 7 6 3 4
13 14^{L} A 7 6 2 12 11 3 4 1 8 5 14 13 A 6 2 1 8
5 13 A 6 7 2 1 8 5 13 14 $A^{\dagger}$ 7 2 1 8 5 6 3 4
9 10 14 13 A 2 7 6 9 10 13 A 2 6 7 3 9 4 10 13
14 $A^{\dagger}$.

122 moves (144).

† = corner position.
23 moves (26) to top right corner
+ 33 moves (40) to bottom right corner
+ 36 moves (43) to bottom left corner
+ 30 moves (35) to top left corner

122 (144)

C68. Moving the small squares from piece A to piece B is denoted below as A^1, A^2, A^3, A^4.

Move definition (i).

10 8 3 2 10 A 7 6 4 10 2 9 8 A 6 7^{L} 4 2 5 10^{L}
2 6 7 A 3 9 8 B 1 2 10 5 B 8 9 3 A A^1 4 6
7 5 4 A 3 9 8 B 4 5 7 6 A 4 B 8 9 3 4 A
6 7 5 B 8 9^{R} 3 4 A 7 5 10 B 9 A 7 5 10 B A^2
9 8 A 5 10 6 B 8 A 5 10 6 B A^3 8 9 A 10 6 7
B 9 A 10 6 7 B A^4 9 8 A 6 7 5 B 9 4 3 B 5
10 2 1 B.

124 moves.

Move definition (ii).

10 8 3 2 10 A 7 6 4 10 2 9 8 A 6 7^{L} 4 2 5 10^{L}
2 6 7 A 3 9 8 B 1 2 10 5 B 8 9 3 A A^1 4 6
10 5 7^{R} 4 A 3 9 8 B 4 5 7^{L} 10 6 A 4 B 8 9 3
4 A 6 10 5 7^{R} B 8 9^{R} 3 4 A 10 6^{L} 7 5 B 9 A 6
7 10 5^{D} B A^2 9 8 A 6 7^{U} 10 5 B 8 A 6 10 7 5^{L} B
A^3 8 9 A 6 10^{R} 7 5 B 9 A 6 7 5 10^{U} B A^4 9 8 A
7 6^{D} 10 5 B 9 4 3 B 5 10 2 1 B.

(144) moves.

C69. Moving the small squares from piece A to piece B is denoted below as A^1, A^2, A^3, A^4.

11 12 A 4 13 12 A 3 2 10 9 11 14 A 3 2 10 1 14 B
7 8 5 6 13 12 A 11 14 B 8 5 6 12 A 14 11 B A^1 8
7 5 6 12 13 A 11 B 7 5 6 13 12 A 11 14 B A^2 7 8
5 6 12 A 14 B 8 7 5 6 12 13 A 14 11 B 7 5 6 13
A 11 14 B A^3 7 8 5 6 13 12 A 14 B 8 6 13 12 A 14
11 B A^4.

103 moves (129).

C70. Moving the small squares from piece A to piece B is denoted below as A^1, A^2, A^3, A^4.

11 12 A 4 13 12 A 3 2 10 9 11 14 A 3 2 10 1 14 B
7 8 5 6 13 12 A 11 14 B 8 5 6 12 A 14 11 B A^1 8
7 5 6 12 13 A 11 B 7 5 6 13 12 A 11 14 B A^2 7 8
5 6 12 A 14 B 8 7 5 6 12 13 A 14 11 B 7 5 6 13
A 11 14 B A^3 7 8 5 6 13 12 A 14 B 8 6 5 12 13 A
14 11 B A^4 8 7 6 5 13 A $11^\dagger$ $14^\dagger$ 1 10 2 3 A 14 11 9
10 2 3 A 12 13 4 A 12 11. 130 moves (164).

† = these pieces effectively 'cross over'.

C71(a).

18 17 16 11 18 17^R 16 15^R 13 A 1 2 13 A 12 14 11 15 7 5
6 18 17 4 3 16 15 7 13 A 12 1 2 16 15 7 13 5 6 18
17 4 3 7 15 16 A 13 5 6 17 4 3 7 15 16^R A 13 12 5
6 17 18 4 3 7 15 16 A 13 2 1 12 5 6 16 A 2 13 16
18 4 3 7 15 17 A 2 16 18 4 3 7 17 A 2 18 4 3 7
17 15 A. 103 moves (121).

C71(b). 6 5 18 11 14 . . . and continue as above in C71(a) from move 6 . . .17, 16, etc. 103 moves (121).

C72.

12 B 5 6 4 3 1 2 A 12 11 B 5 6 4 3 1 2 A 11
B 6 5 A 11 12 B 6 5 A 2 1 11 12 2 1 12 4 3 12
11 4 3 A 8 7 C 12 11 A 8 7 C 11 A 3 4 2 1 7
8 C 11 12 A 3 4 2 1 7 8 C 11 12^U 9 10 11 12 C 6
5 11 12^U 10 9 C 12 11 5 6 11 12^U C 9 10 5 6 11 12 C
5 6 12 C 8 7 B 11 12 C 8 7 B 12 C 6 5 7 8 B
12 11 C 6 5 10 9 7 8 B. 130 moves (136).

C73.

12 B 5 6 4 3 1 2 A 12 11 B 5 6 4 3 1 2 A 11
B 6 5 A 11 12 B 6 5 A 2 1 11 12 2 1 12 4 3 12
11 4 3 A 8 7 C 12 11 A 8 7 C 11 A 3 4 2 1 7
8 C 11 12 A 3 4 2 1 7 8 C 12 A 4 3 C 12 11 A
4 3 C 11 12^R 7 8 12 11 C 3 4 A 7 8 11 12^L 1 2 3
4 A 7 8 C 4 3 1 2 11 C 5 6 B 12 11 C 5 6 B
11 C 1 2 3 4 5 6 B 11 12 C 1 2 3 4 5 6 B 12
140 moves (148).

C74.

9 D 3 4 5 6 10 9 D 3 4 6 9 10 5^R 6 9 10 5 6
A 1 2 6 5 10 9 A 6 5 9 A 4 3 D 10 9 A 4 3
D 9 A 5 6 3 4 D 9 10 A. 51 moves (54).

C75(a).

9 D 3 4 5 6 10 9 D 3 4 5 10 9 6^R 5 A 1 2 9
10^U 6 5 A 1 2 9 C 8 7 5 6 10 9 C 8 7 5 6 9
10^R A 4 3 D 10 9 A 4 3 D 9 A 6 5 B 10 9 A 6
5 B 9 A D 3 4 5 6 B 7 8 C 1 2 4 3 D A 9
10^L 7 8 C 1 2 4 3 5 6 B 9 10 7 8 9 C 2 1 B
10 9 C 2 1 B 9 C 8 7 A 10 9 C 8 7 A 9 C 5
6 D 10 9 C 5 6 D 9 C A 7 8 5 6 D 3 4 B 1
2 8 7 A C 9 10^R 3 4 B 1 2 8 7 5 6 D 9 10 3
4 9 B 1 2 D 10 9 B 1 2 D 10 5 6 7 8 D 10 9
5 6 7 8 D 10 2 1 B 5 6 9 10^D 2 1 B 5 $6^{1/2}$ 10 9
5 4 3 C 10 9 5 6 4 3 C. 211 moves (222).

C75(b). As C75(a) with the following two moves added to the end—9, 10. 213 moves (224).

C76(a). Move definition (i).

1 C A 6 7 5 6 7 A C 2 3 4 6 7 5 8 9 10 11
12 1 2^R C A 5 8 B D 2 C 3 4 A D 2 11 12 10 2
11^R D A 4 3 C 12 D 2 11 9 B 5 8 6 7^R 4 3 C 12
1 D 2 A 6 7 4 3 6 C 1 12 D 2 11 A 7 6 C 12
D 11 A 6 7 C 12 1 D 11 2 A 7 C 1 D 2 11 A 10
9 6 B 8 5 C 7 6 B 8 5 C 6 B 9 10 A 7 6 B
9 10 A 6 B 1 12 D 11 7 6 B 1 12^D C 5 8 9 10 1
12 C 5 8 9 10 12^D C D 11 7 6 B A. 154 moves.

Move definition (ii).

1 C A 6 7 5 6 7 A C 2 3 4 6 7 5 8 9 10 11
12 1 2^R C A 5 8 B D 2 C 3 4 A D 2 1 11 12^L 10
1 2^R D A 4 3 C 11 D 2 1 9 B 5 8 6 7^R 4 3 C
11 12 D 2 A 6 7 4 3 6 C 12 11 D 2 1 A 7 6 C
11 D 1 A 6 7 C 11 12 D 1 2 A 7 C 12 D 2 1 A
10 9 6 B 8 5 C 7 6 B 8 5 C 6 B 9 10 A 7 6
B 9 10 A 6 B 12 11 D 2 1^U 7 6 B 11 12^D C 5 8 9
10 12 11 C 5 8 9 10 11 C D 2 7 1 6^L B A.
(180) moves.

C76(b). Move definition (i).

1 C A 6 7 5 6 7 A C 2 3 4 6 7 5 8 9 10 11
12 1 2^{R} C A 5 8 B D 2 C 3 4 A D 2 11 12 10 2
11^{R} D A 4 3 C 12 D 2 11 9 B 5 8 6 7^{R} 4 3 C 12
1 D 2 A 6 7 4 3 6 C 1 12 D 2 11 A 7 6 C 12
D 11 A 6 7 C 12 1 D 11 2 A 7 C 1 D 2 11 A 10
9 6 B 8 5 C 7 6 B 8 5 C 6 B 9 10 A 11 2 D
7 6 B 9 10 A 11 B 1 6 7 D 2 11 B 1 6^{D} C 5 8
9 10 1 6 C 5 8 9 10 6 C 7 12 D 11 B A 1 6 10
9 5 8 12 7 D 3 4 7 D B 2 11 3 4 7 12 8 5 C
A. 181 moves.

Move definition (ii).

1 C A 6 7 5 6 7 A C 2 3 4 6 7 5 8 9 10 11
12 1 2^{R} C A 5 8 B D 2 C 3 4 A D 2 1 11 12^{L} 10
1 2^{R} D A 4 3 C 11 D 2 1 9 B 5 8 6 7^{R} 4 3 C
11 12 D 2 A 6 7 4 3 6 C 12 11 D 2 1 A 7 6 C
11 D 1 A 6 7 C 11 12 D 1 2 A 7 C 12 D 2 1 A
10 9 6 B 8 5 C 7 6 B 8 5 C 6 B 9 10 A 1 2
D 7 6 B 9 10 A 1 B 12 11 6 7^{U} D 2 1 B 11 12^{D} C
5 8 9 10 12 11 C 5 8 9 10 11 C 6 7 D 1 B A 12
11 10 9 5 8 7 6 D 3 4 6 D B 2 1 3 4 6 7 8
5 C A. (213) moves.

C76(c). Move definition (i).

1 C A 6 7 5 6 7 A C 2 3 4 6 7 5 8 9 10 11
12 1 2^{R} C A 5 8 B D 2 C 3 4 A D 2 1 11 12^{L} 10
$1^{\dagger}$ $2^{\dagger}$ D A 4 3 C 11 D 1 2 9 B 5 8 6 7^{R} 4 3 C
11 12 D 1 A 6 7 4 3 6 C 12 11 D 1 2 A 7 6 C
11 D 2 A 6 7 C 11 12 D 2 1 A 7 C 12 D 1 2 A
10 9 6 B 8 5 C 7 6 B 8 5 C 6 B 9 10 A 2 1
D 7 6 B 9 10 A 2 B 12 11 6 7^{U} D 1 2 B 11 12^{D} C
5 8 9 10 12 11 C 5 8 9 10 11 C 6 7 D 2 B A 12
11 C $6^{\dagger}$ $7^{\dagger}$ D 3 4 5 8 6 7 C 10 9 7 C A 12 11 10
9 7 6 8 5 D B $1\frac{1}{2}$ 2 3 4 5 8 6 7 9 10 11 12^{L} A
C 6 7 8 6 7 C A 12. 209 moves.

† = note that these pieces 'cross over'.

Move definition (ii).

1 C A 6 7 5 6 7 A C 2 3 4 6 7 5 8 9 10 11
12 1 2^{R} C A 5 8 B D 2 C 3 4 A D 2 1 11 12^{L} 10
1 2^{R} D A 4 3 C 11 D 2 1 9 B 5 8 6 7^{R} 4 3 C
11 12 D 1 2^{R} A 6 7 4 3 6 C 12 11 D 1 2 A 7 6
C 11 D 2 A 6 7 C 11 12 D 2 1 A 7 C 12 D 1 2
A 10 9 6 B 8 5 C 7 6 B 8 5 C 6 B 9 10 A 2
1 D 7 6 B 9 10 A 2 B 12 11 6 7^{U} D 1 2 B 11 12^{D}
C 5 8 9 10 12 11 C 5 8 9 10 11 C 6 7 D 2 B A
12 11 C 6 7^{R} D 3 4 5 8 7 6 C 10 9 7 6^{U} C A 12
11 10 9 7 6 8 5 D B $1\frac{1}{2}$ 2 3 4 5 8 6 7 9 10 11
12^{L} A C 6 7 8 6 7 C A 12. (243) moves.

D1.

7^{L} 6 5 4 A 7 6 5 4^{D} 3 B 1 2^{U} 7 A 4 3 7^{D} A 2
1 B A. 23 moves (26).

D2.

7^{L} 6 5 4 A 7 6 5 4^{D} 3 B 1 2^{U} 7 A 4 3 7^{D} A.
19 moves (22).

D3.

7^{L} 6 5 4 A 7 6 5 4^{D} 3 B 1 2^{U} 7 A 4 3 7^{D} B $3\frac{1}{2}$
7 4 A B 1 2 B A 4 3 1 B A 1 2 B A.
37 moves (41).

D4.

6 5 4 A 1 3 4 3 1 A 6 5 4 B 2 3 5 6 4 B
5 6 4^{L} A 1 6 4 A. 28 moves (33).

D5.

12 7 2 3 4 9 8 A 7 2 3 4 A 3 4 A 3 7 17 16
2 4 7 17 16 22 23 18 22 2 21 23 2^{L} 16 17 3 1 A.
38 moves (40).

D6.

10 2 6 5^{DR} $3^{\dagger}$ 2 10 5^{U} 6 7 8 4 $1^{\S}$ 9^{D} 3 8^{U} 4 7 8 $10^{\ddagger}$
2 4 3^{R} 9 1^{LU}. 25 moves (31).

§ = move only one unit; ‡ = move only two units;
† = move one unit right, then down as far as possible.

D7.

2 5 10 6 7 9 3^{D} 9^{L} 6 7^{UL} 4 10 7 6 $4^{\ddagger}$ $3^{\dagger}$ 1 8^{L} 10 $8^{\dagger}$.
20 moves (31).

† = move one unit right, then up as far as possible; ‡ = move up as far as possible, then one unit left and one unit up.

D8.

4UR A 4 2 3 2 BD 1L 2 BR 4UL B 3U C DR 5 6D A C 3
B RD 2 1R 4 2LU C A B 3 D 5R 6UR A C DL 3D B 1 4 2R
CU D† 6‡ 3LU BDL 5 B A. 48 moves (61).

† = left, up, left, up, ‡ = up, left, up, left.

D9. Move definition (i).

10 11 5 2 1 10 11R A 8 9 3 4 12 5 11 10 A 9 3 4
12 13 5 10 11D A 9 8 3 4 11 10 A 9 1 2 6 7 5 13
10 11 4 3 8 9 1 2 7 6 A 4 3 9 1 2 7 6 A 13
10 5 A. 63 moves.

Move definition (ii).

10 11 5 2 1 10 11R A 8 9 3 4 12 5 11 10 A 9 3 4
12 13 5 10 11D A 9 8 3 4 11 10 A 9 1 2 6 7 5 13
12 10 11D 4 3 8 9 1 2 7 6 A 4 3 9 1 2 7 6 A
13 12 5 A. (73) moves.

D10. Move definition (i).

1 13L 14 7 8 6 5 1 13L 12 14 2 8 7 12 1 10 9 3 4
5 10 6 7 12 9 13 14 2 11 14 13 9 1 2 9 13 A 3 4
1 2 9 13 14 A 3 4 1 2 13 12 7 6 10 5 1 2 13 A
11 14 12 A 13 10 9 A 12 8 7 A 9 5 6 A.
76 moves.

Move definition (ii).

1 13L 14 7 8 6 5 1 13L 12 14 2 8 7 12 1 10 9 3 4
5 10 6 7 12 9 13 14 2 11 14 13 9 1 2 9 13 A 3 4
1 2 9 11 13 14L A 3 4 1 2 11 12 7 6 10 5 1 2 11
A 14 13 12 A 11 10 9 A 12 8 7 A 9 5 6 A.
(87) moves.

D11. Move definition (i).

5 12 13U 6 7 4 3 10 9 8 A 5 13 6 7 10 9 3 4 9
10 7 6 11 8 3 4 9 10 7 6 8 12 3 4 9 2 1 A 5
13 11 3 4 9 10 7 6 12 4 3 11 13 5 A 9 10 12 4 3
13 11 8 3 4 12 13 11 A 9 10 11 A 8 3 4 12 6 7 13
12 A 11 1 2 12 A 11 10 1 2 12 13 A 11 6 7 A.
98 moves.

Move definition (ii).

5 12 13U 6 7 4 3 10 11R 9 8 A 5 13 6 7 11 10 3 4
10 11 7 6 9 8 3 4 10 11 7 6 8 12 3 4 10 2 1 A
5 13 9 3 4 10 11 7 6 12 4 3 9 13 5 A 10 11 12 4
3 13 9 8 3 4 12 13 9 A 10 11 9 A 8 3 4 12 6 7
13 12 A 9 1 2 12 A 9 11 1 2 12 13 A 9 6 7 A.
(114) moves.

D12.

A 8 9 10 A 8 9 10 11 5 6 7 13 12 A 11 10 5 6 7
12 A 11 8 1 2 3 4 13 12 A 8 3 4 12 A 8 11 3 4
12 13 A. 43 moves (51).

D13.

13 7 6 5 A 8 9 10 13 7 6 5 11 A 8 9 10 13 7 6
5 11 12 A 13 10 7 6 5 12 A 13 8 1 2 3 4 11 12 A
8 3 4 12 A 8 13 3 4 12 11 A. 52 moves (61).

D14.

13 12 7 6 5 10 11 3 4 13 12 7 6 5 11 3 4 13 2 1
8 9R A 10 11 3 4 9 8 A 11 3 4 8 A 11 10 3 4 8
9 A 11 1 2 13 A 11 10 1 2 13 12 A 11 6 7 A.
58 moves (67).

D15.

12 6 5 11 10 6 13 5 7 4 3 11 10 6 9 8 2 1 A 11
10 3 4 5 7 9 8 2 1 A 10 3 4 7 5 6 A 10 11 3
4 6 A 10 1 2 8 A 10 11 1 2 8 13 A 10 5 9 12 A.
60 moves (69).

D16. Move definition (i).

7 6 5 8 9 12 5 6 7 A 8 9 6 5½ 13 10 7 A 8 9
6 12 5 7 10½ 13 7 A 9 6 12 5 11 7 10 A 9 8 1 2
3 4 13 10 A 8 3 4 10 A 8 9 3 4 10 13 A.
57 moves.

Move definition (ii).

7 6 5 8 11 9 12U 5 6 7 A 8 11 6 5½ 13 10 7 A 8
11 6 9 5 7 10½ 13 7 A 11 6 9 5 12 7 10 A 11 8 1
2 3 4 13 10 A 8 3 4 10 A 8 11 3 4 10 13 A.
(64 moves).

D17.

11 10 12 13 3 4 11 10 12 13 3 4 13 12 2 1 9 3 4 7
6 5 A 8 9 3 4 5 A 9 3 4 5 7 6 A 9 8 3 4
1 2 5 7 6 13½ 12 6 A 8 9 7 5 10 11 12 13 6 A 9
7 5 10 11 6 13 A 10 11 6 13 12 A. 73 moves (85).

D18.

13 12 11 1 2 7 6 5 A 8 9 1 2 5 A 9 1 2 5 7
6 A 9 8 1 2 11 12 3 4 6 7 5 12 11 10 13 3 4 5
11 10 13 3 4 5 6 7 11 A 8 9 12 13 5 6 7 11 10 A
13 12 5 6 7 10 A 12 9 5 6 7 10 11 A.
75 moves (89).

D19. Move definition (i).

13 12 10 2 1 8 9 11 10 2 1 8 9 11 1 2 12 13 4 3
8 9 11 1 2 11 5 6 7 8 9 3 4 13 11 5 6 7 9 3
4 11 5 6 7 9 8 3 4 11 13 5 6 12 10 2 1 A 8 9
3 4 11 6 10 12^{D} 2 1 A 9 3 4 11 13 6 7 A 9 8 3
4 7 A 9 1 2 12 A 9 8 1 2 12 10 A 9 6 5 A.

99 moves.

Move definition (ii).

13 11 12 10^{R} 2 1 8 9 10 12 2 1 8 9 10 1 2 11 13 4
3 8 9 10 1 2 10 5 6 7 8 9 3 4 13 10 5 6 7 9
3 4 10 5 6 7 9 8 3 4 10 13 5 6 11 12 2 1 A 8
9 3 4 10 6 12 11^{D} 2 1 A 9 3 4 10 13 6 7 A 9 8
3 4 7 A 9 1 2 11 A 9 8 1 2 11 12 A 9 6 5 A.

(115) moves.

D20. Move definition (i).

A 8 9 5 6 11 13 12 7 13 12 10 5 6 11 3 4 13 12 7
5 6 11 10 3 4 13 12 7 5 6 10 3 4 12 7 5 6 10 11
3 4 12 13 2 1 A 9 3 4 13 A 9 8 3 4 10 11^{U} 6 5
7 12 13 A 9 1 2 13 A 9 8 1 2 13 12 A 9 5 7 A.

80 moves.

Move definition (ii).

A 8 10 11 6 5 9 11 6 5 7 13 12 5 7 9 3 4 13 12
5 7 6 11 9 3 4 13 12 5 7 6 9 3 4 12 5 7 6 9
11 3 4 12 13 2 1 A 10 3 4 13 A 10 8 3 4 9 11^{U} 6
7 5 12 13 A 10 1 2 13 A 10 8 1 2 13 12 A 10 7 5
A.

(94) moves.

D21. Move definition (i).

8 9 11 5 6 7 A 13 10 11 5 10 13 A 7 6 13 A 3 4
7 6 13 10 A 11 12 8 9 5 A 10 13^{U} 6 7 4 3 11 13 10
6 7 13 11 3 4 11 13 7 6 10 12 3 4 11 13 7 6 10 12^{D}
A 5 9 8 1 2 11 13 4 3 A 10 12 6 7 4 3 A 10 5
8 1 2 13 3 4 7 6 10 12^{D} 5 8 9 1 2 13 11 3 4 7
6 A 13 11 3 4 11 13 A 6 7 13 A 8 9 5 10 12^{U} 6 7
13 11 A 8 7 13 11 A.

128 moves.

Move definition (ii).

8 9 10 11^{U} 5 6 7 A 13 11 9 5 11 13 A 7 6 13 A 3
4 7 6 13 11 A 9 12 10 8 5 A 11 13^{U} 6 7 3 4 9 13
11 6 7 13 9 3 4 9 13 7 6 11 12 3 4 9 13 7 6 11
12^{D} A 5 8 10 1 2 9 13 4 3 A 11 12 6 7 4 3 A 11
5 10 1 2 13 3 4 7 6 11 12^{D} 5 10 8 1 2 13 9 3 4
7 6 A 13 9 3 4 9 13 A 6 7 13 A 10 8 5 11 12^{U} 6
7 13 9 A 10 7 13 9 A.

(150 moves).

D22. Move definition (i).

3 11 10 4 9 12 13 6 12 13 8 5 1 2 6 13 8 5 6 13
8^{D} 5 9 4 11^{R} 10 9 4 7 12 8 5 4 10 9 3 6 9 13 5
8 12 7 10 4 13 5 $8^{½}$ 13 5 9 6 3 4 5 13 12 7 5 8
9 6 8 9 6 2 1 8 9 5 10 11 4 3 A 8 9 5 10 13
12 6 13 12 11 4 3 A 9 5 10 11 A 9 8 5 10 11^{U} 1 2
12 A 9 3 4 6 7 13 12 A 9 11 10 5 8 10 3 4 7 6
12 A 2 1 5 10 3 4 7 6 13 A.

132 moves.

Move definition (ii).

3 11 10 4 9 12 13 6 12 13 8 5 1 2 6 12 13^{L} 8 5 6
12 13^{D} 5 9 4 11^{R} 10 9 4 7 8 13 5 4 10 9 3 6 9 12
5 13 8 7 10 4 12 5 $13^{½}$ 12 5 9 6 3 4 5 12 8 7 5
13 9 6 13 9 6 2 1 13 9 5 10 11 4 3 A 13 9 5 10
12 8 6 12 8 11 4 3 A 9 5 10 11 A 9 13 5 10 11^{U} 1
2 8 A 11 10 5 13 9 11 3 4 6 7 12 8 A 10 11 3 4
7 6 8 A 2 1 5 9 3 4 7 6 12 A.

(147) moves.

D23. Move definition (i).

1 2 3 6 7 4 5 13 11 9 A 13 11 5 4 6 7 3 2 1
8 10 A 13 2 1 10 A 13 11 2 1 10 8 A 13 12 9 5 2
1 8 A 12 13 11 1 2 5 9 11 1 2 8 10 A 13 1 2 10
A 13 12 1 2 9 11 5 10 9 11 2 1 12 13 A 11 9 10 5
2 1 13 A 3 6 7 4 5 10 2 1 13 12 A 9 13 12 1 2
12 13 9 3 6 11^{R} 8 10 12 5 4 11^{R} 6 8 10 12 6 11 4 5
13 9 6 11 4 7 10 12 8 3 6 11 9 13 5 7 10 12 4 5
13 9 11 6 3 4 5 9 11 6 5 10 5 9 11 6 2 1 9 11
5 12 8 4 3 A 9 11 5 12 10 13 6 10 13 8 4 3 A 11
5 12 8 A 11 9 5 12 8 8^{U} 1 2 13 A 11 3 4 6 7 10 13
A 11 8 12 5 9 12 3 4 7 6 13 A 2 1 5 12 3 4 7
6 10 A.

223 moves.

Move definition (ii).

1 2 3 6 7 4 5 13 11 9 A 13 11 5 4 6 7 3 2 1
8 10 A 13 2 1 10 A 13 11 2 1 10 8 A 13 12 11 9^{U} 5
2 1 8 A 13 12 11 9^{L} 1 2 5 9 11 1 2 8 10 A 12 1
2 10 A 12 13 1 2 9 11^{U} 5 10 8^{D} 9 11 2 1 13 12 A 11
10 9 8^{U} 5 2 1 12 A 3 6 7 4 5 10 2 1 12 13 A 11
12 13 1 2 13 12 11 3 6 9^{R} 8 10 13 5 4 9^{R} 6 8 10 13
6 9 4 5 12 11 6 9 4 7 10 13 8 3 6 9 11 12 5 7
10 13 4 5 12 11 9 6 3 4 5 11 9 6 5 10 5 11 9 6
2 1 11 9 5 13 8 4 3 A 11 9 5 13 10 12 6 10 12 8
4 3 A 9 5 13 8 A 9 11 5 13 8^{U} 1 2 12 A 9 3 4
6 7 10 12 A 9 8 13 5 11 13 3 4 7 6 12 A 2 1 5
13 3 4 7 6 10 A.

(257) moves.

D24.

$5^{\dagger}$ 3 2 4 8 A 7 1 $\underline{6}$ $\underline{2}$ 4^{LU} 8^{U} A 7 1 2 6^{D} 3 $\underline{5}^{\dagger}$ 4
$\underline{8}$ A 3^{R} $\underline{4}$ $\underline{8}$ $\underline{A}$. 26 moves (35).

† = move only half way (one unit) on first part of move.

D25.

6 $\underline{2}$ $\underline{3}$ 4 $\underline{A}$ 8 $\underline{7}^{DR}$ 1 $\underline{5}$ $\underline{3}$ 2 $\underline{6}$ $\underline{2}$ $\underline{4}$ $\underline{6}$ A 3 $\underline{5}$ 8 $\underline{7}$
1 5 7^{D} 3 2 4 $\underline{6}$ A $\underline{8}$ 1 7 5 3 2 $\underline{4}$ $\underline{6}$ $\underline{A}$.
37 moves (58).

D26.

$\underline{5}^{\dagger}$ $\underline{6}$ $\underline{3}$ $\underline{5}$ $\underline{6}$ $\underline{3}^{LU}$ 2 7^{R} $\underline{6}^{RD}$ 2 7 4 A 8 1 5 2^{D} $\underline{3}$ $\underline{6}$ $\underline{4}$
A $\underline{8}$ 1 5 2 3 $\underline{6}^{\dagger}$ 4 $\underline{7}$ A 3^{R} $\underline{4}$ $\underline{7}$ $\underline{A}$. 34 moves (49).

† = move only half way (one unit) on first part of move.

D27.

$5^{1/2}$ 2 4 8 A 7 1 3 5^{L} 2 3 5 2 6 $\underline{3}$ 5^{R} 1 7^{U} A 5
8^{D} $4^{1/2}$ 3 $\underline{6}$ 3 4^{L} 8 $\underline{5}^{RU}$ A 7 1 2 3 4 2 $\underline{3}$ 2 4^{L} $\underline{5}$ $\underline{8}^{LU}$
A 7 1 2 3 $\underline{4}^{\dagger}$ A 6 $\underline{8}^{RD}$ $\underline{5}^{DR}$ A. 51 moves (60).

† = move only half way (one unit) on first part of move.

D28.

$\underline{5}^{\dagger}$ 1 2 6 3 A $\underline{7}$ $\underline{4}$ $\underline{2}$ $\underline{3}$ A 7 8 4^{R} 2 1 $\underline{5}^{\dagger}$ 3 $\underline{6}$ A
4 $\underline{8}$ 2 1^{D} $\underline{3}$ $\underline{6}$ $\underline{A}$. 27 moves (38).

† = move only half way (one unit) on first part of move.

D29.

$\underline{5}^{\dagger}$ 4 $\underline{7}$ 6^{L} $\underline{2}$ $3^{2/3}$ A 8 1^{D} $\underline{2}$ $\underline{7}^{\dagger}$ $\underline{3}$ A 8 1 2 6^{D} 4 $\underline{5}^{\dagger}$ 3
$\underline{7}$ A 4^{R} $\underline{3}$ $\underline{7}$ $\underline{A}$. 26 moves (37).

† = move only one unit on first part of move; $^{2/3}$ = move only two-thirds way.

D30.

$\underline{5}^{\dagger}$ 1 $\underline{6}$ 2 4^{R} $\underline{7}^{\ddagger}$ 1 7 2 4 3 A 8 1^{D} 7 2 4 6 $\underline{3}$ $\underline{6}^{RU}$
A 8 1 7 2 4 $\underline{5}^{\dagger}$ 3 $\underline{6}$ A 4 $\underline{3}$ $\underline{6}$ $\underline{A}$. 34 moves (47).

† = move only half way (one unit) on first part of move; ‡ = move as far as possible.

D31.

7 $\underline{5}^{\dagger}$ $\underline{1}$ 3 A 8 2 4^{D} $\underline{6}$ 4 6^{R} 2 A 3 1 $\underline{7}$ $\underline{4}$ $\underline{7}$ A 2
6^{D} A 3 1 $\underline{5}$ 4 $\underline{7}$ A 6 2 8^{R} 3 1 $\underline{4}$ $\underline{7}$ $\underline{A}$.
36 moves (47).

† = move only half way (one unit) on first part of move.

D32.

6 $\underline{1}$ 8 $\underline{4}$ 7^{RD} 3 A $\underline{7}$ A 2 4 3 $\underline{5}$ 1 $\underline{6}$ $\underline{1}$ $\underline{5}^{\dagger}$ $\underline{3}$ $\underline{5}^{\dagger}$ $6^{1/2}$
A $\underline{7}$ 2 $\underline{5}^{D}$ 6^{D} A 1 3 $\underline{8}$ A $\underline{6}$ 5 $\underline{2}$ 7^{D} 1 $\underline{3}$ $\underline{8}$ $\underline{A}$.
38 moves (57).

† = move one unit only on first part of move; $^{2/3}$ = move only two-thirds way.

D33.

5 2 4 6^{D} 5^{D} $\underline{2}$ $\underline{5}^{RU}$ 6 $\underline{4}$ A 8 7 1 A 8 A 7 1^{R} 3 $\underline{4}$
$\underline{6}^{LD}$ $\underline{2}$ $\underline{5}$ $\underline{8}^{LU}$ A 4 7 1^{R} $\underline{6}$ 2 $\underline{5}$ $\underline{8}$ $\underline{A}$. 33 moves (48).

D34.

7 $\underline{6}^{\dagger}$ $\underline{1}$ 8 1^{D} $\underline{7}$ $\underline{6}^{DR}$ 1 8^{U} 3 A 2 3 7 4^{U} 3^{R} 7 $\underline{A}$ 5 8
$\underline{1}$ $\underline{8}$ A $\underline{6}$ $\underline{7}$ $\underline{3}$ 4^{D} $\underline{A}$ $\underline{7}^{\dagger}$ 3 2 5^{D} 1 $\underline{8}^{\dagger}$ $\underline{6}$ $\underline{A}$.
36 moves (52).

† = move one unit only on first part of move.

D35.

5 $\underline{1}$ $\underline{6}$ 3 A 8 2 6^{D} 7 4 $\underline{1}$ 5 1^{U} $\underline{6}^{UL}$ $\underline{4}$ 7 $\underline{2}$ 8 A $\underline{6}$
$\underline{4}$ 6 A 8 $\underline{2}$ 7^{D} $\underline{6}^{RD}$ 1 $\underline{5}$ $\underline{4}$ $\underline{3}$ A 8 2 $\underline{6}$ 1 5^{D} $\underline{4}$ 3 A
8 2 6 7 1 5^{L} 4 $\underline{3}$ $\underline{A}$. 49 moves (67).

D36.

3 $5^{1/2}$ 1 6 2^{U} $\underline{7}$ $\underline{8}$ 1 5^{L} 6 2 7 4 A^{R} 1 $\underline{5}$ 6 $\underline{2}$ 6^{D} $\underline{3}$
$\underline{8}$ $\underline{4}$ A 5 1 $\underline{2}$ 6^{D} 3 $\underline{8}^{\dagger}$ 4 $\underline{7}$ A 3^{R} $\underline{4}$ $\underline{7}$ $\underline{A}$.
36 moves (49).

† = move only half way (one unit) on first part of move.

D37.

5 2 7^{L} 4 A 8 3 6^{L} 8 A 4 $\underline{7}$ 8 6 1 5^{L} $\underline{2}$ $7^{\ddagger}$ $\underline{8}$ 4
$\underline{6}$ $\underline{7}$ 1 $\underline{5}^{\dagger}$ 3 $\underline{A}$ 6 $\underline{7}^{DL}$ 1 $\underline{8}$ 4^{D} $\underline{5}^{RD}$ $\underline{2}$ $\underline{3}$ A $\underline{7}$ 1 5 8 4
2 $\underline{3}$ A $\underline{7}$ 6 1 8 5 4 $\underline{2}$ $\underline{3}$ $\underline{A}$. 52 moves (70).

† = move only half way (one unit) on first part of move; ‡ = move one unit left.

D38.

5 1 6^{R} 3 A $\underline{7}$ 2 4 5^{R} $\underline{1}$ $6^{1/2}$ 3 A 7 8 2 4^{D} $\underline{5}$ $\underline{1}$ 6^{R}
$\underline{3}$ A $\underline{8}$ 2 5 6 4 1 3 A $\underline{8}$ 7 2 5^{D} $\underline{8}$ $\underline{7}$ A 3 1 6^{R}
8 5 4 6 $\underline{5}$ $\underline{8}$ $4^{1/2}$ 6 5 8 1 3 A $\underline{7}$ 4 8 1 $\underline{3}$ $\underline{A}$.
59 moves (74).

D39.

$\underline{5}$ $\underline{1}$ $\underline{7}$ 3 A 8 4 2 1 6^{D} $\underline{5}^{DR}$ 1 $\underline{6}^{LU}$ $\underline{2}$ 4 8^{U} A $\underline{3}$ 7 $\underline{2}$
6 $\underline{1}$ $\underline{6}^{L}$ $\underline{8}$ $\underline{4}$ A 3 7 $\underline{2}$ $\underline{6}$ $1^{1/2}$ $\underline{8}$ $\underline{4}$ $\underline{5}$ A $\underline{2}$ 6^{R} 1 $\underline{4}$ $\underline{5}$
$\underline{A}$. 41 moves (61).

D40.

5 6 1 3 7 A 8 A 4 3 7 8^{L} 2 6^{R} 5 2 4 A 3 8
7 5 2 4 A 3 7 8^{R} 2 5^{D} 1 4 6 A 7 8 2 5^{D} 1 4
$6^{\dagger}$ A. 42 moves (55).

† = move one unit only on second part of move.

D41.

$5^{\dagger}$ 1 3 A 6 7 2 4 1^{R} 5^{D} 1 5 6 A 7 8 4 2 5 4^{D}
6^{RD} 1 3 A 8 2 6 5 4 1 3 A 8 7 2 5 6 4 1 3
A. 41 moves (56).

† = move only half way (one unit) on first part of move.

D42.

6 1 3 7^{LD} 2 $5^{\dagger}$ 4 A 8 7^{DR} 2 5 3 1 6^{L} 4 A 7 2 5
3 1 $6^{\dagger}$ 4 A 7 8 2 7 1 $3^{1/2}$ 5 7 1 6 3 1 6 1 3
4 A. 42 moves (59).

† = move only half way (one unit) on first part of move.

D43.

5 $6^{\dagger}$ 1 7^{L} 4 7^{R} 2 A 3 1 6 4 6 A 2 7^{D} A 3 1 5
4 6 A 7 2 8^{R} 3 1 4 6 A. 31 moves (41).

† = move only half way (one unit) on first part of move.

D44.

I H W X H I A B L^{L} C K Y E T W I A^{U} K Y E
T W I A K H K A I W T E Y H^{D} A K I X B L
C H K I A I K H C L B X A^{U} K H C L B X A
W K H C I Y E T K H W A X B L C I Y T E^{L}
K T E Y I C L B X A C W H T E K Y I^{D} W H
T E K Y I L W H T E C A X B W H T E^{D} A X
B W H T E L I Y K C L A X W H B W H^{L} X A
L C K Y I E T B H W X A^{L} L T B E I Y K C
L T B^{R} A T X W H E I A T B T A I E H W X
B^{U} A I E H W X B L A I T C K Y E T I.
198 moves (237).

D45.

9 1 2^{L} 10 7^{U} 8 4 3^{R} 10 2 13 10^{L} 3 6 12 A 5 11^{U} A 12^{L}
6 5 3 10 13^{D} 2 7 8 4 3 5 6 12 A 11^{R} 13 10 7 8 2
1^{R} 10 7 13 11 A 12^{L} 6 5 8 13 7 10 1 2^{L} 3 13 12 5 8^{D}
13 7 12 7 13 3 4^{L} 8 5 13 7 10 11 A 6 5 13 8 4 3
7 10^{RU} 12 11 A 6^{U} 5 13 8 7^{R} 12^{RU} A 11^{R} 2 1 3 4 7 12^{RU} 10^{RU}
11 A 13 5 6 2 1 3 4^{L} 7 10 11 A 13 2 5 8 13 2 1
3 4 7 10 12 11 A 1 4 10 12 9 A. 133 moves (172).

D46.

9 5 6 10 13 2 1 3 4 11 A 12^{UL} 4 12 8 7 10 3 13 5
6 10^{L} $7^{1/2}$ 8 12 3 13 5 6 10^{UR} 2 1 8 7 10 2 1^{U} 7 8 11
8 7^{L} 13 5 6 2 1 7 8 11^{LD} 13 10 6 5^{U} 3 4 12 A 11^{R} 13
3 4 5 6 1 10 3 8 7 10^{L} 3 4 8 13 11 A 12^{L} 5 6 8
13 7 10 3 4^{L} 1 13 12 6 8^{D} 13 7 12 7 13 1 2^{L} 8 6 13
7 10 11 A 5 6 13 8 2 1 7 10^{RU} 12 11 A 5^{U} 6 13 8 7^{R}
12^{RU} A 11^{R} 4 3 1 2 7 12^{RU} 10^{RU} 11 A 13 6 5 4 3 1 2^{L} 7
10 11 A 13 4 6 8 13 4 3 1 2 7 10 12 11 A 3 2 10
12 9 A. 163 moves (207).

D47.

11 7 $9^{1/2}$ 10 1^{U} 10 8 10 13^{U} 5 6^{DL} 2 12 7 8 9 1 13 5 6
3 A 4 12^{L} 7 2 10^{R} 5 6^{R} 3 12^{LD} 3 5 6 3 5 13 1 9^{L} 6
3 2 $5^{\dagger}$ 13 1 9^{D} 6 3 2 5^{U} 10 7 4 A 12^{R} 13 1 9^{D} 6 3
2 8 7 5^{U} 9 4 9 4 5 10^{U} 1 6 10 5 4 1^{R} 10^{RD} 5^{D} 2 8
7 4 1 10 12 6 5 8 12 10 6 8 5 10 8 7 2 3^{R} 12^{U} 8
13 10 A 9^{L} 5 6 7 13 8 12 3 2^{L} 1 13 9 6 7^{D} 13 8 9
8 13 1 4^{L} 7 6 13 8 12 10 A 5 6 13 7 4 1 8 12^{RU} 9
10 A 5^{U} 6 13 7 8^{R} 9^{RU} A 10^{R} 2 3 1 4 8 9^{RU} 12^{RU} 10 A 13
6 5 2 3 1 4^{L} 8 12 10 A 13 2 6 7 13 2 3 1 4 8
12 9 10 A 3 4 12 9 11 A. 190 moves (246).

† = move two-thirds way.

D48.

9 5 7 6 10 9^{DL} 2 10^{RU} 13 9 10 6 1 8 3 11 A 9^{D} 10^{D} 13
4 9^{RD} 4 10^{R} 13 8 13 10 4 8 9 12 A 11^{R} 3 9^{LU} 11^{UL} 13 $9^{1/2}$ 1
6 7 10 7 9 4 9 2 7 5 6^{R} 1 4 9^{LU} 12 13 12 11 3 11
A 13 9 2 8^{U} 13 9 12 2 12 8 7 12 6^{RD} 5 4 6 5 12^{D} 4
1 6 5 2 12 7 8 12^{R} 2 8 5 6 1 4 7 12 8 2 3 6
5 4 7^{D} 12 7^{U} 8 2 8 7 12^{R} 2 3 8 13 9 A 11^{R} 6 5 8
13 7 12 2 3^{R} 4 13 11 5 8^{D} 13 7 11 7 13 4 1^{R} 8 5 13
7 12 9 A 6 5 13 8 1 4 7 12^{LU} 11 9 A 6^{U} 5 13 8 7^{L}
11^{LU} A 9^{L} 3 2 4 1 7 11^{LU} 12^{LU} 9 A 13 5 6 3 2 4 1^{R} 7
12 9 A 13 3 5 8 13 3 2 4 1 7 12 11 9 A 2 1 12
11 10 A. 203 moves (277).

D49.

11	2	1^{U}	$\underline{19}^{UR}$	20	17	19	20^{R}	1	2	20^{U}	19	$\underline{5}$	8	20	$\underline{19}$	$\underline{2}$	19	20	14
11	20^{U}	11	14^{U}	19	$\underline{2}$	19	8^{U}	$\underline{5}$	19	2	1^{U}	16	5	18	4	12	6	15	3
7	13	11	2	1	16	17^{D}	5	$\underline{19}^{UR}$	$\underline{17}$	$\underline{6}$	12	$\underline{4}$	$\underline{19}$	4	17^{D}	18	8	14	2
$\underline{1}$	$\underline{16}$	14	$\underline{1}$	$\underline{16}$	11	14	5	$\underline{13}$	7	3	15	12	6^{L}	$\underline{17}$	$\underline{22}^{\dagger}$	$18^{\S}$	17^{R}	$\underline{6}$	$\underline{22}$
18^{L}	4	19^{D}	8	1	2	11	14	5	6	$\underline{18}$	$\underline{4}$	$\underline{19}^{LD}$	12	$\underline{22}^{DL}$	$\underline{18}$	$\underline{4}$	$\underline{17}$	16	6
5^{R}	7	3	15	$\underline{18}$	$\underline{4}$	22	18^{D}	15	7	4	3	5	6^{L}	16	$\underline{17}$	$\underline{22}$	3	4^{R}	$\underline{15}$
7	5	$\underline{6}$	16	$\underline{17}$	$\underline{3}$	4	15	18	7	4	3	$\underline{15}$	$\underline{18}$	7	4	$\underline{3}$	$\underline{6}$	$\underline{16}^{\ddagger}$	$\underline{14}$
11	2	1	18	7	12	8^{D}	15	$\underline{17}$	1	2	11	14	16	18	$\underline{7}$	$\underline{22}$	3	4	12
7	18	1	2	15	17^{L}	8	7	$\underline{17}$	8	15	2	1	18^{U}	$\underline{22}$	$\underline{3}$	4	6	5	16
18	3	4	6	$\underline{5}$	$16^{\S}$	18^{L}	$\underline{4}$	5	6^{U}	$\underline{22}$	$\underline{6}$	$\underline{17}$	12	$\underline{22}$	16	18	4	3	17^{U}
$\underline{6}$	$\underline{22}$	6	5^{R}	$\underline{16}$	9	21	A	19^{L}	10	7	15	12	22	8	6	5^{R}	$\underline{16}$	18	9
16	$\underline{18}$	5	6^{L}	8	22	12	15^{U}	7	10	$19^{1/2}$	A	21^{R}	16	18	12	22	12	18	16
21	A	19^{D}	12	$\underline{6}$	5	8	15	7	6	5	8	9	16	18^{U}	8	5	6	10	22
19	12	A	21^{R}	$\underline{8}$	16	18^{D}	9	15	7	10	6	5	18	16	8	21	A	$\underline{19}$	22
12^{D}	6	5^{D}	18	7	10	17	1	2^{D}	11	14	13	3	4^{U}	9	15	8	16	$\underline{21}$	19
A	$22^{\S}$	12	6	5	18	19	A	21^{D}	8	9	4	3	$\underline{13}$	14	11	2	1^{R}	$\underline{17}$	7
18	19	A	8	9	4	3	14	13	$\underline{15}$	16^{U}	7	19	18	A	12	22	21	8	9
4	3	14	13	15	16	7	19	A	12	$\underline{21}^{UR}$	8	9	4	3	7	$\underline{19}^{\dagger}$	$16^{\S}$	15^{L}	$\underline{17}$
19	18	A	12	$\underline{21}$	$\underline{22}$	$\underline{5}$	22	21	12	A	19	18	$\underline{17}$	15	16^{U}	$\underline{19}^{LD}$	16	15^{D}	13
14^{R}	7	3	4	9	8	5	6	21	22^{D}	10	1	2	$\underline{17}$	$\underline{18}$	14	16	15	$\underline{19}$	A
12	5	6^{L}	$\underline{21}$	$\underline{22}$	10	1	2	17	18	14	15	19	A	12	21	22^{U}	6	$\underline{5}^{D}$	9
4	3	7	$\underline{13}$	11	$\underline{14}$	$\underline{18}$	$\underline{15}^{RU}$	$\underline{A}$	$\underline{12}$	$\underline{21}^{\ddagger}$	22	9	16	$\underline{21}^{\P}$	16	22	9	1	2^{L}
17	18	15	A	19^{D}	13	11	14^{L}	$\underline{13}^{UL}$	$\underline{19}$	$\underline{A}$	21	12	$\underline{2}$	18	$17^{1/2}$	1	16	21	12
9	$\underline{3}$	7	16	$\underline{11}$	14	13	$\underline{20}$	13	14	$\underline{20}$	14	13	$\underline{A}$.						

474 moves (568).

† = move only half way (one unit) on first part of move; ‡ = move only half way (one unit) on second part of move; § = move only two-thirds way (two units); ¶ = move only two-thirds way (two units) on second part of move.

D50.

5	7	6	4	9	2	8	4	9	4	8	9^{U}	2	4	7	5	6	A	4	9	2
8	7	5	10	3	7	8	2	4	A	9^{U}	4	2	8	4	5	$10^{\dagger}$	6	9	10^{U}
6	5	$\underline{4}$	8	2	6^{L}	$\underline{10}$	$\underline{4}$	3	$7^{1/2}$	8	$\underline{9}$	3	$\underline{5}$	$\underline{4}$	$\underline{10}^{RU}$	$\underline{3}$	8	7^{D}	$\underline{9}$
$\underline{5}$	$\underline{4}$	9^{R}	$\underline{3}$	9	3	4	10^{R}	3	4	5	7	8	$\underline{4}$	5	7^{L}	10	1	3	7
4	5	8	10	1	3	$\underline{7}$	3	1	10^{U}	8	4^{D}	5^{D}	$\underline{6}$	4	5	9	2^{U}	8	10
1	4	5	3	6^{D}	$\underline{7}$	$\underline{3}$	6	7	3	6	1	10^{U}	5	4	7	10	4	7^{R}	10^{D}
3	$\underline{A}$	6	$\underline{1}$	5	4	3	10	7^{R}	8	2	10	9	A	1	5	4	3	A	$\underline{9}$
10^{U}	2	8	7	3	A^{R}	7	8	2	$\underline{7}$	$\underline{9}^{DL}$	$\underline{10}^{RD}$	$\underline{7}$	A	3	8	$\underline{9}$	$\underline{10}^{DR}$	$\underline{A}$.		

159 moves (186).

† = move only two-thirds way.

E1. The second part of each number refers to the position.
1–6 4–7 12–10 13–14. 4 moves.

E2. All numbers refer to positions, not pieces.
1–5 3–7–1 8–4–3–7 6–2–8–4–3 5–6–2–8 1–5–6 7–1.
7 moves.

E3. All numbers refer to positions, not pieces. The first move is a white piece—the next will be obvious.
1–5 7–1 3–7 8–4–3 2–8 6–2 5–6. 7 moves.

E4.
1 2 3 1 2 6 5 3 1 2 6 5 3 1 2 4 8 7 1 2
4 8 7 4 5 6. 26 moves.

Note: this puzzle is topologically equivalent to that shown in Figure 19.

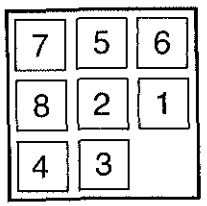


Figure 19

E5. All numbers refer to positions, not pieces.
9–10 6–9 5–6 2–5 1–2 7–1 8–7 9–8 10–9.
9 moves.

E6. All numbers refer to positions, not pieces.
9–10 8–9 7–8 4–7 5–4 6–5 9–6 8–9 7–8 4–7
5–4 2–5 3–2 6–3 9–6 10–9. 16 moves.

E7. A move only counts when a piece moves from the main line to the branch line or vice versa. It is impossible to solve the puzzle in 20 moves which the instructions suggest. 26 moves is the minimum.
2–D 3–D 4–D (1–B) 4–A 3–B 8–D (7–C) 6–D 2–B
7–D (5–C) 7–B 6–B 4–D (2–A) 3–D 8–B (5–B) E–C
5–D 8–D (2–B) 3–B 4–B 6–D 7–D 5–B E–D (1–C)
8–A 6–B 7–B. 26 moves.

Moves in parentheses do not count as no piece moves from or onto the branch line. Although other moves along the main line will be necessary, they are not shown as they will be obvious during the course of the solution.

E8. I N T E R P R E T I N G. 12 moves.

E9. Numbers refer to the 'arms'

J2	I3	J3	H2	G2	F2	E2	D2	C2	B2
A2	J1	K1	L1	I1	A3	B3	C4	D4	E1
F1	G1	H1	B1	A2	B2	D1	C2	D2	H3
G3	F4	E2	F2	G2	H2	I2	L3	K3	J.

40 moves.

E10, E11. Several of the 'buttons' are slightly smaller than the others and/or have an eccentric spindle (which cannot be seen). If the button with the eccentric spindle is placed at the extreme left of the long slot and rotated appropriately, and then two other buttons placed next to it, a fourth one can

then be made to slip down into the lower slot. The rest is then easy. The key letters (or words) are as follows, the first letter being placed on the left:
PERPLEXITY : XIR(Y)
AUTOMOBILE : BOO(T) or TIL(E). Different versions have different words.

E12, E13. These puzzles have an interesting variation on the parity principle. In E12 parity must be changed and therefore an odd number of pieces must go through the corridor. In E13, however, parity does not need to change and an even number of pieces should go through the corridor.

E12.
8 9 5 6 3 2 8 9 5 6 8 9 9† 2 3 5 6 8 5 6
9. 21 moves.

† = this piece comes straight out into the corridor again, having gone into the puzzle from the corridor on the previous move. There is nothing in the rules to say that a piece can not do this. The puzzle can be done in 23 moves without going in and out of the same door as follows:
2 5 9 8 2 5 9 8 2 5 9 3 6 8 2 5 8 6 3 2
5 8 9. 23 moves.

E13.
2 9 4 3 8 1 5 7 9 4 3 8 7 5 2 4 3 5 2 4
3 5 2 9 5 2 9 4 3 2 9 8 7 4 5 6 3 2.
38 moves.

E14. Where there is a choice piece 9 always moves down right unless otherwise stated.
1 3 9^{UL} 2 9^{DL} 6 9 3 9 2 9 6 9^{UL} 4 9 5 9 3 9 2
9 6 9 4 9 5 9 1 9 5. 30 moves.

E15.
22 23 30 31 28 26 19 16 9 10 11 12 3 4 6 13 16 19 26 28
30 31 22 23 16 (out). 25 moves.

E16.
11 12 13 14 15 16 21 25 26 31 29 30 37 A 1 2 4 17 18 A
37 29 30 31 25 26 21 A 17 18 5 6 7 8 15 16 13 14 11 12
A (out). 41 moves.

E17.
36 37 31 32 30 23 24 25 17 18 19 20 11 12 10 5 3 4 9 15
16 5 10 11 12 19 20 17 18 25 23 24 30 33 34 26 27 25 5 15
16 13 14 21 22 28 29 30 5 25 26 27 33 34 31 32 36 37 5
(out). 59 moves.

E18.
$12^{3/4}$ 7 4 5 2 3 4 A 6 8 11 $10^{1/2}$ 1 6 A 9 4 3 2 3
5 12 A (out). 23 moves.
$^{3/4}$ = move three-quarters of the way.

E19. Pieces go to positions indicated by numbers: 1–4 top row (left to right); 5–8 middle row; 9–12 bottom row.
N–5 L–11 B–7 N–1 B–2 ?–5 L–6 K–8 L–12 B–11
K–2 B–6 L–8 B–12 I–6 P–7 B–9 P–12 L–10 P–8
I–12 P–11 ?–8 K–5 N–7 K–1 U–5 E–6 K–4 E–1
N–3 P–2 U–11 P–7 E–5 P–1 U–2 E–11 ?–5 E–6
I–8 E–12 U–11 I–2. 44 moves.

E20. Pieces go to position indicated by numbers: 1–5 top row (left to right); 6–10 middle row; 11–15 bottom row.
T–6 L–14 B–8 T–1 B–2 ?–6 L–8 E–10 L–15 B–14
E–2 B–8 L–10 B–15 I–7 H–8 W–9 B–11 W–15 L–12
H–13 I–14 ?–10 E–8 T–6 E–1 A–9 C–8 K–7 E–5
K–1 T–4 K–6 C–1 A–3 I–2 K–14 I–8 A–6 I–3
A–2 ?–6 K–8 W–10 K–15 A–14 W–2 A–8 H–10 A–13
H–14 ?–10 W–8 C–6 W–1 H–2 C–14. 57 moves.

E21. Moves are shown by giving the position of a piece at the end of each slide, as follows: l, m, r = left, middle, and right columns, respectively, and u, d = up and down, respectively.
4md Bmd Amu 51u 61u A1d 7mu 8mu 3mu Bru
4ru 3ru 2md 1md Ard Crd Dmd 81d 71d 61d
51d Dmu 11u 21u Dmd Cmu 31u 41u Bmu Dru
Bmd Cru Bru. 33 moves.

E22. The engine unable to move is No. 5, and the others move as follows:
7 6 3 7 6 1 2 4 1 3 8 1 3 2 4 3 2.
17 moves.

There are two other slightly different solutions.

E23.
4 1 2 4 1 6 7 1 5 8 1 5 6 7 5 6 4 2 7.
19 moves.

E24.
G Y O Y G P Y O P G O G O Y G P Y O P G
Y P. 22 moves.

E25.
7 5 2 3 6 2 4 7 5 8 2 6 3 4 8 5 7 8 5 2
6 3 4 5 2 7 8 2 5 4 3 6 32 moves.

E26. The numbers refer to those on the left half of the dominoes. Where ambiguity arises the number on the right half of the domino is also given in superscript. A blank is represented by a zero.

4 6 1 4 6 5 4^{0} 6 5 4 6 1 4 6 1 5^{2} 6^{3} 4 6 4
6 1 5 6^{3} 4^{2} 6 4 6 1 5 6 4 6^{0} 4 6 1 5 6 4^{2} 5^{3}
4. 41 moves.

E27. The numbers refer to those on the left half of the dominoes.

4 1 4 5 6 2 5 4 0 6 2 5 4 0 6 2 0 4 3 6
1 6 2 0 5 3 6 4 5 4 6 2 0 5 3. 35 moves.

E28.

B $G^{1/2}$ Y W R <u>Y</u> B Y R W B^{D} Y R W B Y G B R G
B W <u>G</u> R Y^{L} <u>G</u> R W Y G^{D} Y^{D} R^{L} <u>G</u> B $R^{\ddagger}$ W G <u>$W^{\dagger}$</u> <u>B</u> Y.
Then replace the red piece. 40 moves (46).

$^{1/2}$ = move only half way; † = these two pieces 'cross over'; ‡ = red piece reaches top right corner. NB At move 32, slide the *small square* red piece.

E29. All numbers refer to positions, not pieces.

10–5 1–8 11–6 2–9 12–7 3–4 5–12 8–3 6–1 9–10
7–6 4–9 12–7 3–4 1–8 10–5 6–1 9–10 7–2 4–11
8–3 5–12. 22 moves.

E30. All numbers refer to positions, not pieces.

2–3 9–4 10–7 3–8 4–2 7–5 8–6 5–10 6–9 2–5
1–6 6–4 5–3 10–8 4–7 3–2 8–1 7–10. 18 moves.

E31. This puzzle is exactly the same as the one above (E30) except 'doubled up'. The solution is exactly the same except twice as long.
36 moves.

E32. The solution to the checkmate is shown in Figure 20.

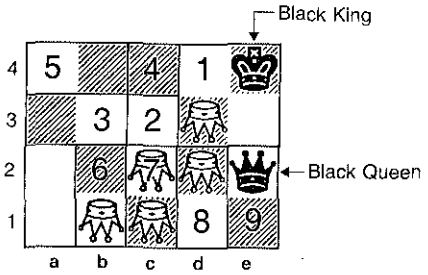


Figure 20

An explanation of the checkmate position, which readers not familiar with Fairy Chess may appreciate is best given by quoting George Jelliss: 'The Grasshopper at b1 guards d3 (over c2). The Grasshopper at c1 guards e3 (over d2). The Grasshopper at c2 gives check (i.e. guards e4 over d3). The Grasshopper at d2 guards d4 (over d3). The Grasshopper at d3 acts purely as a hurdle for the other Grasshoppers. The black Queen cannot stop the mate since the Grasshopper at d2 prevents it capturing the Grasshopper at c2, and if it takes the Grasshopper at d3, this is still check as Grasshoppers can hop over chessmen of either colour.'

Note that if the normal solution for puzzle C19 is used it produces a position with pieces 6 and 7 reversed, and 8 and 9 reversed. This is not a checkmate. Therefore, a solution must be found which not only moves piece 1 to the top right corner, but also leaves pieces 6, 7, 8, 9 in their original start positions.
The full solution is as follows:

6 7 4 2 1 6 7 <u>2</u> 1 3 5 8 9 4 2 1 3 5 8 9
<u>2</u> 1 7 6 3 5 8 9 2 4 1 8 3 5 6 7 8 1 4 2
9 3 5 6 7 8 1 2 4^{L} 9 3 5 6 7 8 1 6 3 5 9
2 4^{R} 6 1 8 7 3 5 9 2 4 6 9 <u>4</u> 3 5 4 2 3 $5^{1/2}$
4 2 3 6 9 1 8 7 2 4^{L} 3 5 6 9 1 8 7 4 2 3
5 8 1 9 6 8 3 5 2 4 7 1 8 6 9 8 1 7 4 2
3 5 6 9 8 1 6 3 5 2 4 7 6 1 8 9 3 5 6 7
4 2 6 $7^{1/2}$ 4 2 6 3 5 1 4 2 6 7 3 5 1 <u>4</u> 8 9
1 4 2 7 6. 165 moves (169).

E33. This type of puzzle can prove enormously difficult unless tackled in the correct way. Therefore, rather than just give a solution, an insight on how to set about finding a solution is described first.

Slots in the pieces are referred to as T, R, A, P, 1, 2 and the small squares that fit into them as T′, R′, A′, P′, 1′, 2′.

1. There are certain arrangements of the pieces where it is possible to exchange (by sliding, of course) the position of two of the smaller squares. One such arrangement is given in Figure 21. In this position the two small squares A′ and T′ can be made to change places.

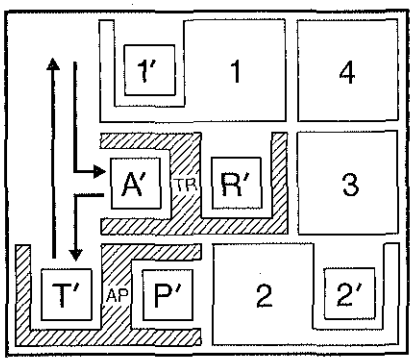


Figure 21

2. Find all the arrangements of the pieces where different pairs of small squares can be exchanged. It will be found that six *different* pairs of exchanges are possible and all are needed to solve the puzzle. The squares in the following pairs of slots can be exchanged: A and T, P and R, T and 1, P and 2, P and 1, T and 2. (Note that either slot P or T is always involved in an exchange.) With the first four of these, there is a unique arrangement of the pieces which will enable an exchange. However, there are six possible arrangements of the pieces which allow the squares in slots P and 1 to be exchanged, and six (mirror image) arrangements for T and 2. Only one of each is necessary to solve the puzzle, unless a minimum move solution is required, when all twelve will need to be found and considered.

Pairs of slots permitting exchanges	No. of arrangements of pieces possible
A T	1
P R	1
T 1	1
P 2	1
P 1	6
T 2	6
	16

3. Find a sequence of exchanges that will put all the small squares into their correct slots. A little experimentation will quickly show that the most difficult squares are A′ and R′. In order to get square R′ from slot A to slot R, it is necessary to move it in turn to slots T, 1 or 2, P and then R. Conversely, square A′ must go through the same sequence in reverse. In theory this means that four exchanges are necessary to move R′ from slot A′ to its 'home' position (slot R) and another four to get A′ home. But in practice putting R′ into its home position automatically removes A′ from its start position. Therefore, a total of seven exchanges will be necessary to put A′ and R′ into their correct places. One of several possible sequences of exchanges is: AT, T1, P1, PR, T2, AT. How many more exchanges are necessary (if any) to put home the other four small squares? The answer is two or four depending on which route was taken in putting R′ and A′ home. A choice of routes was given above offering the alternative of using either slot 1 or 2. If the same slot is used for getting both A′ and R′ home, then four additional exchanges will be necessary. But if a different slot is used for each, then only two additional exchanges are needed. These will be obvious once A′ and R′ are home. Therefore, a minimum of nine exchanges of small squares is necessary to solve the puzzle.

4. From step 3 the order of the exchanges can be derived, and from step 2 the actual arrangement of the pieces necessary to effect an exchange. Now a route (number of moves) must be found to get from the start position to the first exchange position, and then to the next exchange position . . . and finally to the finish or 'solved' position. All positions can be reached from each other in 7–27 moves.
5. Taking this one step further, what is the minimum move solution? To calculate this, it is necessary to go back to 2 above and find the shortest number of moves between all the exchange positions. This is not difficult, except where the squares in slots P and 1 (and also those in T and 2) need to be exchanged. Each has a choice of six exchange positions and the shortest route between the *previous* exchange position and the *subsequent* exchange position must be worked out trying all six in turn. (The alternatives for exchanging T and 2 are mirror images of those for exchanging P and 1.) A further complication is that the two additional exchanges (not involving A′ and R′) can be done either before, after, or half before and half after the other seven exchanges. It will be found that there are several different solutions with 179 moves, but it is quite difficult to find a solution in 177.

The full solution is:

4 A 2 1 T 3 4^{U} A 2 4 3 T $1\frac{1}{2}$ 4 3 T R′ P′ R′ 1
4 3 T^{R} 1 4 3 T 2 A 1 2 T 3 4 2 $T\frac{1}{2}$ 3 4 2 2′
R′ 2′ T 3 4 2^{R} T 3 4 A 1 3 4^{D} A T′ R′ T′ A T 2
A T 3 4^{U} 1 T A $2\frac{1}{2}$ 3 4 A A′ R′ A′ 2 3 4 A^{L} 2 3
4 A 1 T 2 1 A 4 3 1 $A\frac{1}{2}$ 4 3 1 1′ A′ 1′ A 4 3
1^{L} A 4 3 T 2 3 4^{D} T 2′ A′ 2′ T A 1 T A 3 4^{U} 2
A T $1\frac{1}{2}$ 3 4 T P′ A′ P′ 1 4 3 T^{R} 1 4 3 T 2 A 1
2 T 3 4 2 $T\frac{1}{2}$ 3 4 2 T′ P′ T′ 2 T 3 A 1 T A 1′
P′ 1′ 1 T A 1 3 4^{D} 2 1 A T 3 4^{D} A T 3.

177 moves (196).

E34. If the number of pieces in the puzzle is odd, then the first piece to be moved (No. 1) should go into the 'leg' in which you wish all the pieces to finish. If the number of pieces is even, then the second piece to be moved (No. 2) should go into the leg in which you wish all the pieces to finish. Which leg a piece should go into will be obvious except in the case of piece No. 1. The answer is that piece 1 always goes into every leg in turn and always in the same sequence.

1 2 1 3 1 2 1 4 1 2 1 3 1 2 1 5 1 2 1 3
1 2 1 4 1 2 1 3 1 2 1 6 1 2 1 3 1 2 1 4
1 2 1 3 1 2 1 5 1 2 1 3 1 2 1 4 1 2 1 3
1 2 1 7 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1 5
1 2 1 3 1 2 1 4 1 2 1 3 1 2 1 6 1 2 1 3
1 2 1 4 1 2 1 3 1 2 1 5 1 2 1 3 1 2 1 4
1 2 1 3 1 2 1. 127 moves.

E35. A paper entitled: 'Some results on the Panex Puzzle' written by Mark Manasse and Danny Sleator at AT & T Bell Laboratories (USA) and Victor Wei of Bell Communications Research (USA) gives much interesting data. The paper gives a proof that two columns of pieces can be exchanged in 31 537 moves. (It seems likely that this figure may also prove to be the minimum). So far, however, they have proved only that it requires at least 27 564 moves. The minimum number of moves to transfer one column of pieces to the centre leg is 4 875 and this can be done without disturbing the other column of pieces.

F1. Each figure in the solution represents *two* moves: the numbered half-hexagon plus its blank counterpart (in whichever order is appropriate). A† after a number means that the numbered half is moved without its blank counterpart, and a − means that only the blank half opposite that number is moved.

7 4 1 5 2 3 6 3 5 1 7 2 7 1 7^{-} $3^{\dagger}$ $7^{\dagger}$ 3 1 3
2 3 4 3 2 1 5 7 6 7 6 8 1 6 5 6 4 6 5 8
4 5. 81 moves.

Note. It might be thought that parity principles could not apply to this puzzle. But, as in the 'Fifteen' puzzle (see B10), only half of all random start positions are solvable—unless some 'trickery' is used. Normal moves will consist in moving a complete hexagon (two pieces) into a complete hexagon space. The puzzle is not solvable using only this type of move, because parity needs to be changed and normal moves can not achieve this. Therefore, the trick that is used is to exchange only the numbered halves of two pieces having the same orientation: e.g. 3 and 7 (as in the solution given above) or 1 and 8. The effect of 7^{-} $3^{\dagger}$ $7^{\dagger}$ 3 (above) is to exchange the places of the numbered 3 and 7 whilst leaving their blank counterparts unchanged.

F2, F3. No solutions needed.

F4.

3 4 5 6 3 4 5 2 1 10 4 5 2 6 3 2 5 4 1 6
3 2 7 9 4 1 5 7 9 4 1 8 1 4 9 2 3 7 5 8
4 9 2 3 7 6 10 8 4 5 6 7 3 6 5 4 10 7 5 4
6 3 5 4 6 9 10 9. 68 moves.

F5. P = Protection; F = Free Trade; Protection START bay = 1; Unmarked bay = 2; United Kingdom of Great Britain & Ireland bay = 3; Free Trade bay = 4.

Move P to 2; move F to channel at widest part (just outside bays 3 & 4); turn F 180° so that its concave part faces outwards: move F to 1; rotate P so that its concave surface faces inwards (when going round the channel); move P round channel to 3.

For explanation of the symbols in Puzzles F6–F12 refer to 'Notation' at the beginning of the solutions section.

F6.

7 UN 1 $\overleftrightarrow{2}$ 7 3 4 $\overleftrightarrow{5}$ 6 $\overleftrightarrow{5}$ UN 7 3 4 6 5 UN
17 moves.

F7.

$\overrightarrow{14}$ $\overrightarrow{13}$ 12 11 10 $\overleftarrow{13}$ 11 12 $\overleftarrow{14}$ $\overleftarrow{6}$ 5 4 $\overleftarrow{3}$ 7 8 9 13 11^{U} 12^{U} 13
4 5 $\overleftarrow{3}$ $\overrightarrow{6}$ 24 moves.

F8.

A 10 15 11 10 A 4 8 7 1 7 2 A. 13 moves.

F9. No solution given.

F10.

7 5 6^{D} 2 3^{D} 1 4 2 3^{L} 1 4 2 3^{U} T 7 5 6 1 4 3
2 T 7 5 6 1 7 5 6 1 7 4 $\overrightarrow{2}$ T 5 $\overrightarrow{6}$ 4 $\overrightarrow{2}$ 3 T
7 1 4 $\overleftarrow{6}$ 4 7 2 3 1. 49 moves.

F11.

13 14 12 11 12 14 13 10 $\overleftarrow{12}$ $\overrightarrow{6}$ 7 8 $4^{\dagger}$ 8^{L} 4^{L} $\overleftarrow{3}$ 7 $\overrightarrow{3^{\ddagger}}$ 4^{R} 8^{R}
$\overleftarrow{2^{\parallel}}$ 1 5 1^{L} 6 2^{U} 4^{U} 8^{U} 9 1 5 6 2 4 8^{L} 3 7^{L} 11 9 5
1 6 2 4 8^{L} 7^{R} 3^{R} 5 1 9 11 7 3^{R} 1 5 9 $\underline{8}^{D}$ $\underline{4}^{D}$ $\overrightarrow{2}$ 4^{U}
8^{U} $\overleftarrow{6}$ T 10 $\overleftarrow{12}$ 13 14 11 3 9 1 13 14^{L} $\overleftarrow{11}$ $\overleftarrow{3}$ 14 13 9 1 3^{L}
7 9 1 5 13 14 3^{U} 7^{D} 14 3^{D} 9 1 5 13 4 $\overrightarrow{8^{\S}}$ 2 6 T 10
$\overrightarrow{12}$ 14 7 3 11 9 1 5 4 8^{D} 2 13 6^{R} T 10 12 3 7^{U} 11 13
14 3^{D} 7^{D} 5 8 4 $2^{\P}$ 6 T 10 12^{R} 131 moves.

† = on top of 8; ‡ = under 7; § = left of 4; ‖ = right of 6; ¶ = under T.

F12.

21 22 23 20 19^{D} 18 20 18^{L} 19 23 22 21 17 20 18 19 15 10 14 13
9 8 3 4 2 7 4^{U} 12 3 18 19 15 10 13 14 8^{D} 9^{D} 5 2 8
13 14 9^{L} 5^{D} 2^{D} 8 13 9^{U} 14^{U} 5 2 19 18 15 10^{U} 16 T 17 20 21
22 $5^{2/3}$ 2 23 18 19 10 15 16 21 22 23 2 $5^{\ddagger}$ 18 19 10 15^{D} 16 3
4 12 7^{D} 6^{U} 4 3 7 12 21 22 11 T 17 20 23^{L} $2^{\dagger}$ $5^{\dagger}$ 18 19 21
22 15^{L} 10^{L} 16 13 14 8 9^{D} 6 3 12 4^{D} 7^{D} 1 11 1^{L} 7 4^{U} 12 3^{U}
15 10^{U} 21 22 18 19 16 13 14^{D} 8 9^{D} 10 15 3 12 4 7^{D} 11 1^{R} T
17 20^{U} 2 5^{U} 23 18 19 21 22 9 8 4 7 1 11 6 15 10 12 3^{U}
7 4^{U} 13 14^{U} 8 9 1 11 6 10 15^{D} 12 3 7 4^{U} 13 14^{U} 8^{U} 9^{U} 1^{D}
11 6 10 15^{D} 3 12^{D} 7 4 13 14 8 9^{U} 11 6 3 12 15 10 3 4
6 11 8 13 9^{R} 14^{R} 7 12^{R} T 17 2 5 20^{L} 10 15^{L} 3 4^{L} 6 11 8
13^{L} 9 14 7 12 T 2 17 20 5 15^{U} 10^{U} 3 4^{L} 6 11 8 13 9^{L} 14^{L}
12 7 T 2 17^{R} 245 moves.

† = left, up, left; $^{2/3}$ = two-thirds way; ‡ = to bottom right corner.

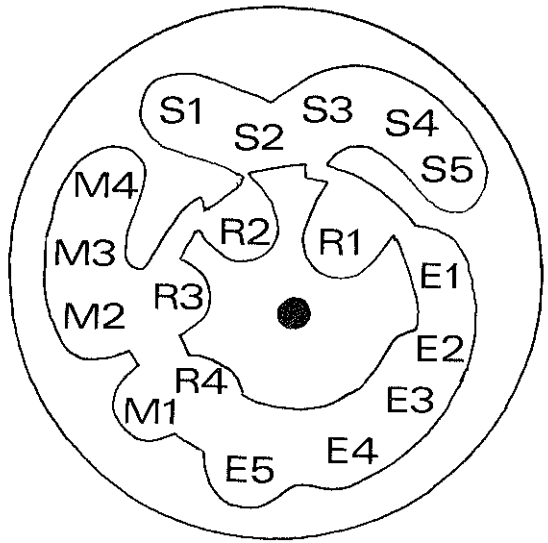


Figure 22

Figure 22 gives all 18 relevant positions for the counters: five start positions S1–S5; five end positions E1–E5; four middle positions M1–M4; and four

rotor positions R1–R4. Counters are Y (yellow), R (red), W (white), B (blue), G (green)

1. Pick up R in R2 and put R at M2.
2. Pick up Y in R2 and pick up R in R1.
3. Put Y at M4. Move W from S3 to S1.
4. (Keeping R in R1) pick up B in R2 and put B at M3 and R at M2.
5. Pick up W in R2, move G from S5 to S1 and put W at S5.
6. Pick up R in R2 and put R at S4.
7. Pick up G in R2 and put G at M2.
8. Pick up R in R2, move W from S5 to S1 and leave R at S5.
9. Pick up W in R2, pick up G in R1 and put W at M2.
10. (Keeping G in R1) pick up R in R2 and put R at M1.
11. (Keeping G in R1) pick up R in R3 and pick up W in R2.
12. (Keeping G in R1) put R at E5 and put W at M1.
13. (Keeping G in R1) pick up R in R4, W in R3 and B in R2.
14. (Keeping G in R1) put R at E1, W at E5 and B at M1.
15. (Keeping G in R1) pick up W in R4, B in R3 and Y in R2.
16. (Keeping B in R3 and Y in R2) put W at E2 and put G at M2.
17. Then put B at E5 and put Y at M1.
18. Pick up B in R4, Y in R3 and G in R2.
19. Put B at E3, Y at E5 and G at M1.
20. Pick up Y in R4 and G in R3.
21. Put Y at E4 and G at E5.

G1. Start with the letters SUMMER on the front facing upwards and WINTER on the back. Move blocks according to the letters that can be seen from the front or the back. Ignore letters that are hidden from view. Pushing a piece through the puzzle from top to bottom or vice versa (i.e. changing a piece from one level to the other) counts as a move.

T E R R E U E T M M $U^{\dagger}$ N R E N R T N I W
S U M T W M U S I W T $U^{\ddagger}$ A $I^{\ddagger}$ S A U U R U
N I I U U R I I R T U. 51 moves.

† = move upwards, not through the puzzle changing levels; ‡ = move downwards.

For those who wish to number the pieces a solution is given below in numerical form.

SS = 1; WA = 2; UP = 3; IT = 4; MN = 5; MI = 6;
TU = 7; EG = 8; EN = 9; RR = 10; RU = 11.

7 9 11 10 8 11 9 7 5 6 11 9 10 8 9 10 7 5 4 2
1 3 6 4 2 6 3 1 6 2 4 3 2 6 1 2 3 11 10 7
5 4 6 3 11 10 4 6 10 4 7. 51 moves.

G2. General procedure for solving puzzles of this type.

1. Find out where the vacant space should be (if possible). Turn the whole cube so that this space is in the top level.

2. Put block 1 in place.
3. Put block 2 in place without disturbing block 1.
4. Put block 3 into the place where block 4 should go.
5. Put block 4 immediately above block 3.
6. Now blocks 3 and 4 will readily go into their proper positions.
7. The remaining three blocks can now easily be arranged into their proper positions.

G3. The same procedure as given for the previous puzzle (G2), suitably extended, can be used to solve the bottom and middle layers. The only real problem arises with the top layer. It will be found that if suitable blocks from the top layer are 'parked' temporarily in the middle layer they can be retrieved so as to put them into the correct order in the top layer. By using one, or even two different parking bays, this type of puzzle readily yields to being solved. At first the biggest difficulty arises out of remembering which block goes where—this can be harder than actually solving the puzzle!

G4. Similar methods to those used for solving the previous puzzle (G3) can be applied. Because of the large number of black beads, the puzzle would appear to be quite easy, but this is far from true. The best order of solving is to solve four faces, which can be done without too much difficulty and then the last two faces (which should be adjacent). By a process of plaiting or interweaving it is possible to position the remaining seven colours:

1. Put all the coloured beads into one face (not in the common edge).
2. Feed three correctly coloured beads one at a time, with the correct number of black beads in between, into the other (second) face until they are all correctly placed.
3. Now put the remaining four coloured beads into the second face, two on each side immediately adjacent to the first face.
4. It should now be a relatively simple matter to feed the last four coloured beads one at a time into the first face with the appropriate black beads in between.

G5. No solution.

For the meaning of the notation in puzzles H1–H10, refer to the beginning of the solutions section.

H1.
④L^2 ②U^1 ③D^1 ⑥$R^{15}D^{1*}$ ⑤R^{18*} ③$D^2R^{14}U^1R^{1*}$
④$L^1D^3R^{14}U^{1*}$ ①$D^5R^{13}D^1R^{1*}$ ②$D^2L^2D^3R^{14*}$. 9 moves (22).

H2.
④R^1 ⑥L^1 ④U^2 ②L^1 ③$D^1L^5U^{3*}$ ①$L^1D^2L^5U^{2*}$
②$L^3D^2L^5U^{1*}$ ④$U^1L^5D^2L^{5*}$ ⑩$U^5L^5D^2L^4D^1L^{1*}$ ⑨R^2
⑧R^2 ⑦$R^2U^3L^4U^{3*}$ ⑧$L^4R^1U^3L^4U^{2*}$ ⑨$L^6R^1U^3L^4U^{1*}$
⑥$D^1L^5R^1U^3L^{4*}$ ⑤$D^1L^4R^1U^3L^3D^1L^{1*}$. 16 moves (53).

H3.
④D^1 ⑩$L^7U^1L^{1*}$ ⑨R^1 ⑪$L^{10}D^1L^{1*}$ ⑨L^2 ④$D^3L^{11}U^{1*}$
⑥$L^1D^3L^{9*}$ ⑦R^1 ⑧$D^1L^7D^1L^{1*}$ ⑦$D^2L^8U^{1*}$
⑨$L^3D^1L^{6*}$ ③R^1 ⑤$D^3L^6D^1L^{1*}$ ①$D^5L^6U^{1*}$
③$L^1D^4L^{6*}$ ②$D^2L^2D^3L^5D^1L^{1*}$. 16 moves (43).

H4.
⑯L^3 ⑧R^5 ⑦R^1 ②R^7U^{2*} ①R^9U^{1*} ⑨U^2R^{8*}
⑦$U^1R^6D^2R^{1*}$ ⑭$U^3R^6D^1R^{1*}$ ⑧$R^2U^2R^6U^{2*}$ ⑬$U^3R^8U^{1*}$
③$D^1R^8U^2R^{6*}$ ④$D^1R^7U^2R^5D^2R^{1*}$ ⑤$D^1R^6U^2R^5D^1R^{1*}$
⑩$U^1R^8U^2R^5U^{2*}$ ⑪$L^1U^1R^8U^2R^5U^{1*}$ ⑥$D^1L^3R^7U^2R^{5*}$
⑫$U^1L^3R^7U^2R^4D^2R^{1*}$ ⑲$U^5R^8U^2R^4D^1R^{1*}$ ⑳$L^1U^5R^8U^2R^4U^{2*}$
⑮$L^1D^3U^5R^8U^2R^4U^{1*}$ ⑯$R^4L^1U^4R^8U^2R^{4*}$
⑰$U^1R^4L^1U^4R^8U^2R^3D^2R^{1*}$ ⑱$U^1R^3L^1U^4R^8U^2R^3D^1R^{1*}$.
23 moves (106).

H5.
④U^1 ⑥D^1 ⑤L^8U^{1*} ⑧U^1L^{8*} ⑦$L^{10}D^1L^{1*}$ ⑨$U^1L^{10}D^{1*}$
⑥$L^1U^1L^{8*}$ ⑩$U^2L^8U^1L^{1*}$ ⑪$R^1U^3L^8U^{1*}$ ⑫$L^1U^3L^{8*}$
④$D^5L^2U^3L^7D^1L^{1*}$ ③$L^1D^1L^5U^{1*}$ ①$D^1L^2D^3L^{5*}$ ②$L^5D^3L^4D^1L^{1*}$.
14 moves (45).

H6.
⑧D^1 ③L^1 ④D^1 ②D^1 ①L^7D^{2*} ②$U^1L^7D^{2*}$ ③$U^1L^7D^{1*}$
④$U^3L^7D^{1*}$ ⑥U^3L^{7*} ⑧$U^5L^6U^2L^{2*}$ ⑩$U^5L^5U^1L^{2*}$
⑦$U^4L^3U^2L^{1*}$ ⑤$R^1U^3L^3U^1L^{1*}$ ⑨$U^2R^1U^3L^{4*}$. 14 moves (38).

H7.
②R^4 ⑤D^4L^{1*} ①R^1 ②D^{6*} ①$L^1D^6R^1D^{1*}$ ③$L^1D^4U^1R^{1*}$
④R^1 ⑥$R^1U^3R^6$ ⑩U^1 ⑪L^1 ⑧D^1 ⑨$R^3D^1R^{1*}$ ⑧U^1L^{6*}
⑦L^6D^{1*} ⑩$R^1U^1R^6D^{1*}$ ⑪U^2R^{7*} ④$L^1D^3R^{5*}$ ⑥D^{3*}.
18 moves (37).

H8.
⑬R^1 ⑮$R^1D^7L^{3*}$ ⑭U^2 ①L^4 ⑦L^2 ⑨$D^3R^1D^7L^{2*}$
④$D^5R^1D^7L^{1*}$ ⑩$D^2R^2D^{7*}$ ⑦$D^4R^2D^6R^2D^{1*}$
①$D^6R^2D^6R^1D^{1*}$ ⑪$L^1D^2R^1D^6L^{3*}$ ⑫$L^3D^2R^1D^6L^{2*}$
⑬$D^1L^1D^6L^{1*}$ ⑤$L^1D^5L^1D^{6*}$ ⑧$L^1D^4L^1D^5R^2D^{1*}$
⑭$D^1L^2D^2L^1D^5R^1D^{1*}$ ⑯$L^4D^5L^{3*}$ ⑰$L^6D^5L^{2*}$ ⑱$L^8D^5L^{1*}$
⑥R^1 ②$L^8D^6R^2D^{5*}$ ③$L^{10}D^6R^2D^4R^2D^{1*}$ ⑥$U^1L^{10}D^6R^2D^4R^1D^{1*}$.
23 moves (86).

H9.

⑨R^4 ⑮R^2D^3 ⑥D^2 ④L^1 ②$U^4R^7U^{2*}$ ⑦$U^6R^7U^{1*}$
⑤$L^2U^5R^{7*}$ ③$L^1U^4R^6D^2R^{1*}$ ④$R^1U^5R^6D^1R^{1*}$ ①$R^1U^4R^6U^{2*}$
⑬$U^8R^6U^{1*}$ ⑭$L^2U^8R^{6*}$ ⑩U^1 ⑨$L^2U^7R^5D^2R^{1*}$
⑫$U^4R^1U^4R^5D^1R^{1*}$ ⑧$R^5U^7R^5U^{2*}$ ⑥$U^1L^3R^5U^7R^5U^{1*}$
⑪$L^6U^7R^{5*}$ ⑩$D^1R^3L^6U^7R^4D^2R^{1*}$ ⑮$U^8L^1U^4R^4D^1R^{1*}$.

20 moves (72).

H10.

④D^1 ⑤R^2 ⑩D^3 ⑨D^1 ㉗D^1 ㉓R^1 ②$U^3R^5D^{10}R^{3*}$
③$L^1U^3R^5D^9R^{3*}$ ⑰$U^3L^1U^3R^5D^8R^{3*}$ ⑫R^1 ⑯$U^6R^5D^7R^{3*}$
⑫$U^2L^1U^3R^5D^6R^{3*}$ ①$R^1U^3R^5D^5R^{3*}$ ⑧$R^1U^4R^5D^{10}R^{2*}$
⑭$U^1R^3U^5R^5D^{10}R^{1*}$ ⑮$R^2U^6R^5D^{10*}$ ⑥D^1 ⑦$R^3U^4R^5D^9L^1D^{1*}$
㉖$U^{11}R^5D^9R^{2*}$ ㉓$L^1U^{10}R^5D^9R^{1*}$ ㉜$U^3L^1U^{10}R^5D^{9*}$
㉒$R^2U^{10}R^5D^8L^1D^{1*}$ ㉘$R^2U^{12}R^5D^8R^{2*}$ ㉔$U^1R^3U^{10}R^5D^8R^{1*}$
㉕$R^2U^{11}R^5D^{8*}$ ㉛$U^3R^1U^{10}R^5D^7L^1D^{1*}$ ㉚$U^3R^2U^{10}R^5D^7R^{2*}$
㉙$R^3U^{13}R^5D^7R^{1*}$ ㉑$L^1D^3U^{11}R^5D^{7*}$ ⑱$R^2U^7R^5D^6L^1D^{1*}$
⑳$R^1D^3U^{11}R^5D^6R^{2*}$ ⑲$R^3D^3U^{11}R^5D^6R^{1*}$ ⑤L^1 ⑪D^1
⑤$R^7U^4R^5D^{6*}$ ⑪$U^2R^6U^4R^5D^5L^1D^{1*}$ ⑥$U^1L^3R^7U^4R^5D^5R^{2*}$
④$R^3U^1R^6U^4R^5D^5R^{1*}$ ⑨$U^2R^8U^4R^5D^{5*}$ ⑬$R^3U^2R^6U^4R^5D^4L^1D^{1*}$
㉗$L^2U^{12}R^5D^4R^{2*}$ ㉝$U^1L^2U^{12}R^5D^4R^{1*}$ ㉞$L^3U^{13}R^5D^{4*}$
⑩$L^1U^4R^8U^4R^5D^3L^1D^{1*}$. 44 moves (198).

J1. The number of changes of direction (moves) is given at the end of each line in parentheses.

E goes to R and connects to B (1).
E takes B via R to L and leaves it (2).
E pushes A back into S and uncouples (1).
E goes via L and R to pick up A at S again (3).
E, A go via R to pick up B (2).
E, A, B go via R into S leaving B (2).
E takes A to B's original place (1).
E goes via R and L to pick up B in S (2).
E leaves B in A's original place and returns to L R (2). 16 moves in total.

There are several ways of solving this puzzle, but most others use 17 or more moves.

J2. Moves (reversals of engine direction) are in parentheses at the end of each line.

(a) E pushes 3 to S and returns to R (1).
E pushes 1 to L and then couples with 2 (2).
E pushes 2 to S and goes to R (1).
E goes to L and then back to S where it couples with 2 (2).

E pulls 2 to its original position, uncouples and couples with 1 at L (1).
E pulls 1 to its original position, uncouples and goes to R (1).
E picks up 3 at S, drops it at its original position and goes to R (2).
10 moves in total.

(b) E picks up 3 and returns to R (1).
E pushes 1 and 3 to L where it leaves them both (1).
E goes to R, S and then pushes 2 into L (3).
E goes to R, S and back to L (3).
E pulls 2 back to its original position (1).
E goes to S, R and L coupling with both 3 and 1 (2).
E leaves 1 in its original position and goes on to R (1).
E pushes 3 back to its original position and returns to R (2).
14 moves in total.

J3.

E couples to 4 and leaves it at M.
E goes via R, S to L and couples to 5.
E pulls 5 to M, uncouples and pushes 4 to R.
E goes via S to L and couples to 6.
E pulls 6 to M, uncouples and pushes 5 to R.
E goes via S to L and couples to 7.
E pulls 7 to M, uncouples and pushes 6 to R.
E goes via S to L and couples to 8.
E pulls 8 to M, uncouples and pushes 7 to R.
E goes via S, L to M and pushes 8 to R.
E pulls 8, 7, 6, 5, 4, 1 to L and reverses back to P where it leaves 1.
E takes 8, 7, 6, 5, 4 to L and back to R where it uncouples.
E goes via L to pick up 1 at P.
E takes 1 via L to R.
E takes 1, 8, 7, 6, 5, 4, 2, 3 to L and reverses to P, S where it leaves 2, 3.
E takes 1, 8, 7, 6, 5, 4 to L and back to R where it uncouples all except 1.
E, 1 go via L to P, S, couple to 2, 3, and return to L.

J4.

F (without coaches) to S.
E, 1, 2, 3 to R.
F to R.
F, 3, 2, 1 to L.
E to S.
F 3, 2, 1 to R.
F 3, 2, 1, a, b, c, d to L.
E to R to L.
E d, c, b, a, 1 to R to S.
E leaves 1 in S.
E d, c, b, a to R to L.
E to R to S.
E, 1 to R to L.
E, 1, d, c, b, a, 2 to R to S.
E leaves 2 in S.
E, 1, d, c, b, a to R to L.
E, 1 to R to S.
E, 1, 2 to R to L.
E, 1, 2, d, c, b, a, 3 to R to S.
E leaves 3 in S.
E, 1, 2, d, c, b, a to R to L.
E, 1, 2 to R to S.
E, 1, 2, 3 to R.

J5.
E moves to d, leaves 4 coaches, goes to b and back via c to a.
F moves via c to a, back via d to b pushing the four coaches with it.
E moves to d.
F moves to c, leaves the four coaches and goes on to a.
E goes to b, backs up to collect the four coaches at c and returns to b.

J6. Divide train 1 into three sections: engine plus the front seven wagons (a), the next eight wagons (b) and the last wagon (c). Train 2 remains intact throughout the solution. Moves (reversals) are given in parentheses at the end of each line.
(a) moves forward slightly to allow Train 2 to push (b) and (c) back to L (0).
(a) proceeds to R (0).
Train 2 pulls (b) into S where it leaves it and goes on to R (1).
Train 2 goes via M to L and couples to (c) (1).
Train 2 goes into S leaving (c) and pushing (b) into R (1).
Train 2 can now proceed through M to L and on its way (1).
(a) and (b), back into S to pick up (c) and return to R (2).
6 moves in total.

J7.
A1 goes to R and then backs into S, pushing the freight cars to make room.
B1, B2 go to L and couple up A2.
Meanwhile A1 goes on to the end of R.
B1, B2 pull A2 to R, reverse and push A2 into S.
B1, B2 uncouple A2, go to R and then on to L.
A1 can now back up to collect A2 from S and return to R.

J8.
F goes to L and then backs up into S with 7, 8, 9.
E takes 1, 2, 3 to M, leaves them and goes on to R.
F goes to L.
E pushes 7, 8, 9 from S to L.
E goes via S to R and then pushes 1, 2, 3 to L.
E goes to R, picks up 4, 5, 6 takes them to M where it leaves them before going on to L.
E takes 1, 2, 3 through S and out to R.
F pushes 7, 8, 9 back into S and returns to L.
F picks up 4, 5, 6 at M and goes out to L.

J9.
E goes into S and couples to 4.
E takes 4 via R to L and uncouples.
E returns to S via R and couples to 3.
E takes 3 via R to M and leaves it there.

E goes via R into S and pushes 2 to L where it leaves it.
E goes via S to R, couples 1 and takes it to S.
E uncouples and goes via L to M.
E pushes 3 to R where it leaves it.
E picks up 1 in S and, going via L, leaves it at M.
E goes via R, S to pick up 2 at L.
E takes L to S and leaves it in its original position.
E goes via R to pick up 1 at M.
E takes 1 via R and leaves it in S.
E goes via R to pick up 4 at L.
E takes 4 via R and leaves it in its original position.
E returns to its own original position.

J10.
E collects 2 at R and goes via L to the bridge where it uncouples.
E goes to L, couples 1, goes to R and then to the bridge where it couples 2.
E takes both 1 and 2 to R and L where it leaves 2.
E goes back to R and up to the bridge where it leaves 1.
E goes via R, M, L and picks up 1 on the other side of the bridge.
E takes 1 via L to R and then returns by itself to M.

J11. Moves (changes of engine direction) are given in parentheses at the end of each line.
E goes to L, couples up to 1, and takes it via D to R (1).
E leaves 1 at T, goes to R, round through D, L, U, and back to R (3).
E pushes 2 to T, and couples up (0).
E pulls 1, 2 to R, and reverses back to D, where it leaves 1 (2).
E takes 2 to R, and pushes it back to T, where it leaves it (2).
E goes through R, U, L, and picks up 1 at D (1).
E pushes 1 to R, and uncouples (0).
E picks up 2 in T, proceeds to R, and back through D to L (3).
E uncouples and returns to S (1). 13 moves in total.

J12. The solution is given in terms of pieces on the turntable before and after each rotation. The movement of the pieces along the track between rotations of the turntable will be obvious. Therefore 14–41 means that pieces 1 and 4 (left, right) are rotated to become 4 and 1. Single numbers mean that a piece is rotated by itself on the turntable.
4, 5, 14–41, 15–51, 16–61, 17–71, 24–42, 34–43. 25–52, 35–53, 26–62, 27–72, 36–63, 37–73, 6, 7. 16 moves in total.

J13. The position of the pieces is shown in relation to the turntable (// //). Each rotation of the turntable (move) is shown by R. The solution given is for the Shunting puzzle. The extra moves (rotations) required for the Turntable Train puzzle are shown by (r) after →. It might be thought that because the track is not in a straight line that the Turntable Train puzzle

requires double the number of moves. But where the turntable has only two pieces on it instead of three, extra moves are not required.

			6543//21E//	R
6543//E12//	→		6543//E1//2	R
6543//1E//2	→		6543//1E2//	R
6543//2E1//	→	(r)	654//32E//1	R
654//E23//1	→	(r)	65//4E2//31	R
65//2E4//31	→	(r)	6//52E//431	R
6//E25//431	→	(r)	//6E2//5431	R
//2E6//5431	→	(r)	2//E65//431	R
2//56E//431	→	(r)	25//6E4//31	R
25//4E6//31	→	(r)	254//E63//1	R
254//36E//1	→	(r)	2543//6E1//	R
2543//1E6//	→		2543//1E//6	R
2543//E1//6	→		254//3E1//6	R
254//1E3//6	→	(r)	25//41E//36	R
25//E14//36	→	(r)	2//5E1//436	R
2//1E5//436	→	(r)	21//E54//36	R
21//45E//36	→	(r)	214//5E3//6	R
214//3E5//6	→	(r)	21//43E//56	R
21//E34//56	→	(r)	//21E//3456	R
//E12//3456				

19 moves.

APPENDIX A: A MATHEMATICAL NOTE

David Singmaster (Series Editor)

The mathematics of sliding piece puzzles either is moderately well known and somewhat complex, or is almost completely unknown. Consequently it has not been discussed in this book. Here I outline it and provide references for those who want to go further. [See section (d) in Appendix C: References and Bibliography.]

There are basically two problems that one wants to solve:

A. Can you get from one configuration to another?

B. What is the least number of moves required to get there?

Problem A is theoretically solved only for certain puzzles with uniform pieces.

1. The basic sliding block puzzle can be generalized to any network of points and lines with pieces placed on all but one point and a move consisting of sliding a piece along a line to the unoccupied point. Such puzzles have been analysed by R. Wilson [8], and the results are summarised in [2].

2. The numerous puzzles inspired by Rubik's Cube can generally be readily analysed by techniques similar to those used for Rubik's Cube. See [5, 3, 1] for the basic ideas and [4, 6, 7] for various further puzzles.

3. The shunting puzzle with a string of cars on a main line and a single siding is equivalent to an input data stream with a storage stack. Such systems have been studied by computer scientists.

In all other cases, Problem A is not solved, though it can be solved practically by solving Problem B as described below.

No method is known for solving Problem B other than systematic searching. However, the classical tree, back-track, or depth-first searching technique does not apply easily. A sequence of moves can bring us back to the initial configuration. The presence of such loops means that our search tree is *not* a tree and compensating for this is elaborate. It is conceptually simpler to do a breadth-first search. Here we generate all the possible configurations of the puzzle and say that two configurations are joined if you can get from one to the other by a single move—using your favourite definition of move. This gives a structure called a graph or a network and there are well known algorithms for determining the minimum number of moves between configurations in this graph. This also determines whether you can get between configurations at all, giving a brute-force solution of Problem A.

I have examined the most common type of 4 by 5 puzzle with 10 pieces and I found about 40 000 possible configurations. The graph-theoretic algorithms can handle such a problem, but we do not know of anyone who has successfully programmed any sliding piece puzzle, much less produced any general program for them. We will be happy to hear from anyone who does so.

APPENDIX B: A LIST OF PATENTS FROM THE PATENT OFFICES OF THE UNITED STATES OF AMERICA AND THE UNITED KINGDOM

Letters and numbers in the first column denote the type of patent as follows:
A capital letter and number refers directly to a puzzle in the text.
a = Patent of a sliding block puzzle mentioned in Chapter 2.
b = Patent of a sliding block puzzle that refers to the construction, rather than to the design, of a particular puzzle.
c = Patent of a miscellaneous design for a sliding block puzzle not referred to in the text.
d = Patent of a railway shunting puzzle.

US Patents

a	91 737	1869	E. E. Gilbert
a	207 124	1878	E. U. Kinsey
a	227 159	1880	M. T. Foote
b	278 571	1883	J. U. MacKenzie
b	286 395	1883	W. J. Decker
d	332 211	1885	P. Protheroe
E34(a)	347 596	1886	A. B. Harris
b	364 465	1887	J. W. Ashborn
c	390 784	1888	J. F. Jones
c	413 756	1889	M. N. Jones
G2(a)	416 344	1889	C. I. Rice
c	422 388	1890	J. H. Flanagan
c	427 392	1890	A. T. Bradshaw
A9(a)	433 444	1890	J. W. Brown
d	437 186	1890	A. G. Farwell
A6	437 932	1890	T. W. McGrath
d	442 445	1890	C. W. Lurtey
c	446 513	1891	T. A. Frazer
c	465 147	1891	W. Trumbull
b	476 980	1892	W. H. Cook
c	481 589	1892	A. C. Proctor

d	482 957	1892	J. A. Allen
c	483 276	1892	P. W. Anderson
c	483 318	1892	C. E. Clarke
c	483 912	1892	C. S. Ford, Jr.
c	498 639	1893	J. C. Fields
a	509 934	1893	C. A. McFadden Jr.
c	510 904	1893	J. B. Douglas
C1(a)	516,035	1894	H. Walton
a	526 544	1894	L. P. Shriver
c	535 279	1895	S. J. Eymann
c	552 721	1896	T. F. Schofield
F13(a)	560 197	1896	W. E. Dow
a	564 022	1896	R. J. Murphy
c	564 846	1896	A. E. Schmidt
a	590 093	1897	D. du Bois
A6	602 735	1898	F. H. Donaldson
A4	604 248	1898	F. D. Hopley
c	605 191	1898	E. A. Shields
c	614 988	1898	N. H. Sanborn
c	618 871	1899	J. S. Hanson
a	619 804	1899	O. Svanström
c	631 737	1899	L. Compton and H. M. Houck
c	638 391	1899	W. Hazell
c	639 602	1899	D. McGenniss
E10	642 374	1900	R. M. Schaffer
c	643 470	1900	J. C. Teller
c	653 290	1900	C. T. Dukes
C14(a)	668 386	1901	F. E. Moss
c	687 153	1901	W. K. Hawks
d	688 339	1901	J. M. Rodgers
d	703 076	1902	F. L. Napier
c	704 089	1902	J. H. O'Bern
d	729 522	1903	M. H. Anderson
b	730 026	1903	A. Keedell
c	737 899	1903	J. Bigham
E7	743 015	1903	E. S. Mowry (filed 1902)
c	743 469	1903	G. Denison
d	750 862	1904	H. Keeler
d	753 266	1904	O. L. Hubbard
D5(a)	771 514	1904	W. H. E. Wehner
c	782 594	1905	W. Brown
d	783 589	1905	L. B. Pennell
G3	785 665	1905	A. Coe
c	795 487	1905	C. W. Courtney
b	807 113	1905	F. O. Dyer

F7(a)	811 321	1906	E. C. Pfeiffer
d	822 862	1906	W. E. McGraw and A. M. Goodale
d	856 749	1907	H. G. Webster
d	878 187	1908	S. S. Cahill
c	897 089	1908	C. B. Graves
d	898 321	1908	J. W. Clark
c	908 410	1908	E. G. Jensen
c	920 462	1909	E. C. Hinkle
C5	922 002	1909	L. C. Koehler and W. M. Butler
c	930 151	1909	W. Brown
c	961 892	1910	E. M. Peck
a	979 923	1910	B. A. Boore and J. W. Ferguson
c	991 963	1911	H. Endinger
C43(a)	1 017 752	1912	L. W. Hardy (filed 1907)
d	1 018 879	1912	C. M. Conley
c	1 082 460	1913	W. M. Bushfield
c	1 085 050	1914	R. Lathrop
c	1 085 994	1914	J. C. Patterson
c	1 090 245	1914	W. Stephens
c	1 101 567	1914	C. L. Ridgway
C2	1 112 746	1914	J. I. Wiley
c	1 132 430	1915	J. A. Brogan and M. Bruner
c	1 144 799	1915	J. J. Tiernan
d	1 174 219	1916	T. F. Young
c	1 174 506	1916	J. L. Kesner
c	1 191 135	1916	J. H. Moore
c	1 206 054	1916	C. A. Trull
c	1 251 822	1918	J. Prendergast
c	1 253 891	1918	H. Schlirf
c	1 257 205	1918	A. C. Fallace
c	1 260 541	1918	G. Holland
b	1 274 294	1918	F. Lobl
d	1 275 210	1918	J. W. Braunschweiler
c	1 277 391	1918	C. L. Crouse
c	1 289 471	1918	M. Jones
c	1 309 659	1919	W. H. Smiley
c	1 336 541	1920	O. J. Shepherd
c	1 349 456	1920	V. A. Hlasko
c	1 364 290	1921	A. G. E. Lowman
d	1 377 039	1921	J. V. Wells
C48, C49	1 415 625	1922	M. Bullister
c	1 438 746	1922	S. H. Colen
b	1 459 937	1923	H. C. Teipel
c	1 464 424	1923	L. H. Hartman
b	1 477 371	1923	F. Larabee

F2, F3	1 495 576	1924	A. C. Crehore
c	1 518 889	1924	W. W. Wooster
c	1 520 666	1924	C. F. Dietz
c	1 538 768	1925	O. E. Wheaton
c	1 550 552	1925	J. R. McMahon
c	1 555 980	1925	G. H. Johnson
d	1 560 921	1925	B. von Bültzingslöwen
D1	1 633 397	1927	C. L. A. Diamond
c	1 637 602	1927	D. L. Chandler
c	1 652 115	1927	L. B. Gottert
C56	1 663 568	1928	J. M. Schneider (filed 1927)
c	1 676 505	1928	T. J. Scanlon
D12,D14,D15	1 683 014	1928	F. L. Babcock
B45	1 879 571	1932	H. W. Spaulding
c	1 899 177	1933	C. F. Beddell
c	1 987 951	1935	H. J. Vander Heide
F9	1 993 211	1935	F. G. Dustin
c	2 007 530	1935	J. Greene
c	2 779 598	1957	I. H. Steinhardt
b	2 788 975	1957	J. F. Lichtenberger
c	2 948 535	1960	J. Ellman
d	3 127 175	1964	L. J. White
F10, F11, F12	3 208 753	1965	S. E. Stotts
G5	3 216 558	1965	O. T. Marsh
G2	3 841 638	1974	F. W. Sinden
G3	3 845 959	1974	D. Kosarek
a	4 036 503	1977	M. L. Golick
B34	4 097 049	1978	M. F. DeVos

UK Patents

d	15 051	1885	?
c	12 748	1889	F. W. Forbes
E34	20 672	1890	A. Gartner and G. Talcott
c	5 337	1901	W. P. Thompson
c	21 815	1901	D. James and A. Pearson
c	18 734	1903	J. Bigham
d	7 647	1905	?
C2	110 186	1916	R. W. Hunter and J. C. Wilson
c	121 691	1918	J. Povey
d	125 350	1918	J. Bell and W. H. Buchan
d	145 289	1919	F. Roberts
d	150 621	1920	G. B. Brinkley
c	199 270	1922	F. C. L. and R. G. Toye
d	232 899	1925	J. H. Farrer

b	245 004	1925	C. Haelbig
c	246 667	1925	P. F. Campbell
C19	381 813	1932	I. I. Isowitsky
b	387 221	1932	M. M. Carpmael
b	403 280	1933	R. R. Reynolds
C27(a)	411 515	1932	J. H. Fleming
a	411 916	1932	M. Bennett
b	417 143	1933	R. R. Reynolds
b	424 870	1934	A. Ritzmann
b	435 660	1934	C. F. Gaunt
b	435 718	1934	C. F. Gaunt
c	577 646	1944	W. Brown
c	596 015	1945	G. W. C. MacLennan
C27	592 536	1946	H. B. Saunders
c	645 120	1948	E. A. H. Deavin, J. A. Mansell and T. Armitage

APPENDIX C: REFERENCES AND BIBLIOGRAPHY

This is divided into four parts:

(a) Books and papers about the '15' puzzle
(b) Books and papers containing other sliding block puzzles
(c) Books and papers containing railway shunting puzzles
(d) References in Singmaster's *Mathematical note* (Appendix A).

In a few cases a book will deal with two or more categories; if so, it is repeated under the relevant headings.

(a) Books and papers about the '15' puzzle

Ahrens, W. (1918). *Mathematische Unterhaltungen und Spiel* (2nd edn), Vol. 2. Teubner, Leipzig.

Austin, A. K. (1979). The 14–15 puzzle. *Mathematical Gazette* **63** (March), 45–6.

Bain, G.G. (1907). The prince of puzzle-makers—an interview with Sam Loyd. *Strand Magazine* **34** (204), 771–7.

Botermans, J. and Slocum, J. (1986). *Puzzles old and new, how to make and solve them*. University of Washington Press, Seattle, USA.

Burbank, D.T. (1938). Letters from Readers. *Games Digest* June, 54–5.

Cramer, B. (1880?). *Lösung aller möglichen Stellungen des Spiele der Funfzehn (Boss Puzzle)*. Leipzig.

Davies, A. L. (1970). Rotating the Fifteen puzzle. *Mathematical Gazette* **54** (389), 237–40.

Domoryad, A.P. (1963). *Mathematical games and pastimes* 79–85. Pergamon, Oxford.

Dudeney, H.E. (1926). The psychology of puzzle crazes. *The Nineteenth Century and After* **100** (6, December), 868–79.

Fleury, H. (1880). *La clé du taquin, ou la solution des quinze*. Marseilles.

Fraser, P. (1880). Three methods and forty-eight solutions of the Fifteen Problem. *Proceedings of the American Philosophical Society* **18** (1878–80), 505–10.

Gardner, M. (1959). *The Scientific American book of mathematical puzzle and diversions*, p. 88. Simon & Schuster, New York.

Gardner, M. (1970). Mathematical games. *Scientific American* **222**, 112–16.

——(1975). *Sixth Book of Mathematical Games from Scientific American* pp. 64–5. Freeman, San Francisco.

Henry, C. (1880). Sur le taquin. *Gazette Anecdotique, Litéraire, Artistique et Bibliographique* **5** 58–9, 87–92.

Johnson, W.W. (1879). Notes on the '15' puzzle 1. *American Journal of Mathematics* **2**, 397–9.

Kirkman, Rev. T.P. (1880). Mathematical questions and solutions: 6489. *Educational Times Reprints* **34**, 113–14.

——(1881). Mathematical questions and solutions: note on the solution of the 15-puzzle in Question 6489. *Educational Times Reprints* **35**, 29–30.

Kowalewski, G. (1921). *Boss-Puzzle und verwandte Spiele*. Koehler, Leipzig.

——(1930). *Alte und neue mathematische Spiele* pp. 61–81. Teubner, Leipzig.

Kraitchik, M. (1943). *Mathematical Recreations* pp. 302–8. George Allen & Unwin, London.

Leopold, L. (1880). *Das System und die Lösung des Boss Puzzle, Spiel der Fünfzehn*. Hamburg.

Liebeck, H. (1971). Some generalizations of the 14–15 puzzle. *Mathematics Magazine* **44** (September), 185–9.

Loyd, S. (1914). The 14–15 in puzzleland. *Cyclopedia of 5000 puzzles, tricks and conundrums* pp. 235, 371. Lamb, New York. (Reprinted in 1976 by Pinnacle Books, New York).

——(1928). *Sam Loyd and his puzzles—an autobiographical review* pp. 1–4. Barse, New York.

——(1959). *Mathematical puzzles of Sam Loyd* (ed. M. Gardner.) Vol. 1, pp. 19–20, 128. Dover, New York.

——(1968). Introduction by Peter van Note. In *The Eighth book of Tan* Dover, New York. (Republication of a work originally published by Loyd & Co. in 1903).

Lucas, E. (1881). Sixième Récréation—sur le jeu du Taquin ou du Casse-Tête Americain. *Revue Scientifique de la France et de l'Étranger* **3** (27), 783–8.

——(1882–94). *Récréation mathématiques* (2nd edn), Vols. 1, 2, 3, 4. Blanchard, France. (Reprinted 1977.)

Meen, G. and Reece (1881). *The fifteen puzzle* (song). Music sheet, Charles Sheard, London.

Mondésir, P. de (1880). Le dernier mot du Taquin. *La Nature* **8** (2nd term), 266–7.

Proctor, R.A. (1881a). The Fifteen Puzzle. *The Gentleman's Magazine* **250** (1801), 30–45.

——(1881b). *Knowledge* 11 November, 37–8.

——(1881c). *Knowledge* 25 November, 79.

——(1881d). *Knowledge* 30 December, 185.

Punch, 8 January (1881). The Fifteen Puzzle song, capitally sung by Mr. Dallas, is deservedly encored.

Redon, P. (1894). *Nouvelles recherches sur le jeu du Taquin*. Paris.

Rouse Ball, W.W. (1892). *Mathematical recreations and problems* (1st edn) pp. 74–8, Macmillan, London.
——(1939). *Mathematical recreations and essays* (11th edn) pp. 299–300. Macmillan, London.
——(1974). *Mathematical recreations and essays* (12th edn) pp. 312–14. University of Toronto Press, Toronto, Canada.
Sala, G.A. (1880). Echoes of the Week. In *Illustrated London News* **76** (2138), 491.
Schubert, H. (1880). The Boss Puzzle. *Hamburgischer Correspondent* **82**.
——(1880). *Theoretische Entscheidung über das Boss. Puzzle Spiel* (2nd edn). Hamburg.
——(1909). *Mathematische Mussestunden* Vol. 2, pp. 133–58. Göschen, Leipzig.
Snowdon, J.W. (1880). The 'Fifteen' Puzzle. *Leisure Hour* **29**, 493–5.
Spitznagel, E.E. Jr (1967). A new look at the Fifteen Puzzle. *Mathematics Magazine* **40** (Sept./Oct.) 171–4.
Story, W.E. (1879). Notes on the '15' puzzle 2; and: (Editorial) notes on the preceding paper. *American Journal of Mathematics* **2**, 399–404, 201–4.
Tait, P.G. (1880). Note on the theory of the '15' puzzle and: Remarks on the previous communication. *Royal Society of Edinburgh Proceedings* **10**, 664–5, 729.
——(1884). Introductory address to the Edinburgh Mathematical Society (9 November 1883)—Listing's topologie. *Philosophical Magazine* (5), 17, pp. 30–46, Scotland.
Tissandier, G. (1880). Les carrés magiques à propos du Taquin—jeu mathématique. *La Nature* **8** (2nd term), 81–2.
Warren, G.W. (1880). Letter to the Editor: clew to the Fifteen Puzzle. *The Nation* April (774), 326.

(b) Books and papers containing other sliding block puzzles

Barnard, D. St. D. (1973). *Figure it out*. Pan, London.
Berlekamp, E.R., Conway, J.H., and Guy, R.K. (1982). *Winning ways* Vol. 2, pp. 769–77; 810. Academic Press, New York.
Botermans, J. and Slocum, J. (1986). *Puzzles old and new, how to make and solve them*. University of Washington Press, Seattle, USA.
Dawson, T.R. (1928). *Original puzzles, puzzle no. 101*. Manuscript in British Chess Problem Library, England.
Dudeney, H.E. (1902). The Canterbury puzzles. In *The London Magazine* **48** (July), 597–600.
——(1903). A budget of puzzles. In *Strand Magazine* **26** (150), 739.
——(1907; 1919). *The Canterbury Puzzles*, Nelson, London. (Reprinted in 1958 by Dover, New York.)
——(1908). World's best puzzles. *Strand Magazine* 779–87.

——(1910). Perplexities. In *Strand Magazine* **233** (May), 628.
——(1917). *Amusements in mathematics*. Nelson, London. (Reprinted in 1958 by Dover, New York.)
——(1926). *Modern puzzles*. Pearson, London.
——(1931). *Puzzles and curious problems*.
——(1967). *536 puzzles and curious problems*. Scribner, New York.
Filipiak, A. (1942). *100 puzzles: how to make and solve them*. Barnes, New York. Reprinted (1978) as *Mathematical puzzles and other brain twisters*. Bell, New York.
Gardner, M. (1959). *Scientific American book of mathematical puzzles and diversions*. Simon & Schuster, New York.
——(1970). Mathematical games. *Scientific American*, February.
——(1975). *Sixth book of mathematical games from Scientific American* pp. 64–70, 200–1, 206–7. Freeman, San Francisco.
Hughes, F. (Clark) (1947). Block puzzle. In *Amateur Handicraft Magazine*. pp. 58–9. Bruce, Milwaukee.
Lešan, P. (1927). *Početní a jiné hříčka zabarg*. Czechoslovakia.
Loyd, S. (1914). *Cyclopedia of 5000 puzzles, tricks and conundrums*. Lamb, New York. (Reprinted in 1976 by Pinnacle Books, New York.)
——(1928). *Sam Loyd and his puzzles—an autobiographical review* pp. 17–18; 89. Barse, New York.
——(1959). *Mathematical puzzles of Sam Loyd* (ed. M. Gardner) Vol. 1. Dover, New York.
Millington, R. (1979). *Games and puzzles for addicts*, pp. 49, 96. M.&J. Hobbs in association with Michael Joseph, London.
Reilly, W.H. (1949). In *Fairy Chess Review* 7, 85.
Rubin, D. (1979). *What's the big idea = The puzzle book*. Lippincott/Black, USA/London.
——(1980). *TWA Ambassador Magazine* May, pp. 132–4.
——(1984). *The real puzzle book*. World Almanac, New York.
Schwarz, Benjamin L. (1973). A new sliding block puzzle. *Mathematics Teacher* March, 277–80.
Singmaster, D. (1982). *Cubic Circular*. Issue 2, pp. 9–13. Singmaster, London.
Smith, A.L. (1940). Change the seasons. *Popular Science Magazine* July, 183.
——(1942). Strategy. *Popular Science Magazine* September, 229.
Sperling, W. (dates unknown). *Die Grubelkiste; Kuriuse Probleme; Amusante Zeitrertrieb; 1000 kleine Wunder; Die verhexte Westenache; Spiel und Spass furs ganze Jahr; Zauberei mit Karten*—formerly *Denkspiele fur kluge Kopke*.
Truran, T. (1978). Solid gold. *Games and Puzzles Magazine* Winter (71), 6–7.
——(1981). Solid gold. *Games and Puzzles Magazine*. Summer (81), 38.
Weiss, H. (1977). *Games and puzzles you can make yourself*, Hamish Hamilton.

Wells, K. (1971). The fun of it. *Popular Mechanics Magazine* May, p. 117.
Wyatt, E.M. (1956). *Puzzles in wood* pp. 14–15. Bruce, Milwaukee.
Yoshigahara, N. (1985). In *Quark Visual Science Magazine* May, 45; 143.
Anon. (1938). Quite easily done? Some geometrical puzzles. *Games Digest* August, 17–19; October, 53–4.
——(1944). *Industrial Arts and Vocational Education Magazine* September, 284.
——(1932). *Popular Science Magazine* October, p. 91.
——(1933). *Popular Science Magazine* October, pp. 76–7.
——(1981). In *Top Puzzles Magazine*. June/July, p. 39.
——(1981). In *Top Puzzles Magazine*. July/August, p. 42.
——(1981). In *Top Puzzles Magazine*. October/November, p. 44.
——(1944). Wit sharpeners. (A collection of puzzles, pranks and problems—Any questions? Series—No. 2). *Padiham Advertiser*, Burnley, England.

(c) Books and papers containing railway shunting puzzles

Coxeter, H.S.M. and Rouse Ball, W.W. (1974). *Mathematical recreations and essays* (12th edn) pp. 116–8. Toronto Press, Toronto.
Degrazia, J. (1949). *Maths is fun* pp. 65–8; 132–3. George Allen & Unwin, London.
Dudeney, H.E. (1917). *Amusements in Mathematics* pp. 62; 194–5. Nelson, London (Reprinted in 1950 by Dover, New York.)
——(1925). *The world's best puzzles. Daily News*, London.
——(1967). *536 puzzles and curious problems* pp. 134, 339. Scribner, New York.
Kraitchik, Maurice (1943). *Mathematical recreations* pp. 222–6. George Allen & Unwin, London.
Loyd, S. (1914). *Cyclopedia of 5000 puzzles, tricks and conundrums* pp. 89, 167, 350, 361. Lamb, New York (Reprinted in 1976 by Pinnacle Books, New York.)
——(1928). *Sam Loyd and his puzzles—an autobiographical review* pp. 54, 106. Barse, New York.
——(1959). *Mathematical puzzles of Sam Loyd* (ed. M. Gardner) Vol. 1, pp. 92, 155. Dover, New York.
——(1960). *Mathematical puzzles of Sam Loyd* (ed. M. Gardner) Vol. 2, pp. 18–19, 128. Dover, New York.
Lucas, E. (1883). *Récréations mathématiques* Vol. 2, pp. 131–4. Blanchard, Paris.
Rouse Ball, W.W. (1892). *Mathematical recreations and problems* (1st edn) pp. 43–4. Macmillan, London.
Sainte-Laguë, A. (1942). *Avec des nombres et des lignes* (2nd edn), pp. 16–30. Vuibert, Paris.

References

(d) References in Singmaster's *Mathematical note.*

1. Bandelow, C. (1982). *Inside Rubik's Cube and beyond*. Birkhäuser, Boston.
2. Berlekamp, E.R., Conway, J.H., and Guy, R.K. (1982). *Winning ways*. Academic Press, London.
3. Frey, A.H. Jr., and Singmaster D. (1982). *Handbook of cubik math*. Enslow Publishers, Hillside, NJ.
4. Rubik, E. *et al.* (In press). *Rubik's cubic compendium*. Oxford Univ. Press, Oxford.
5. Singmaster, D. (1980) *Notes on Rubik's "Magic Cube"*. 5th edn, published by the author, London; Enslow Publishers, Hillside, NJ (1981); Penguin, London (1981).
6. Singmaster, D. (1981–85) *Cubic Circulars* 1–8. Published by the author, London.
7. Singmaster, D. (1984) Hungarian Rings groups. *Bull. Inst. Math. Appl.* **20**, 137–9.
8. Wilson, R. (1974) Graph puzzles, homotopy and the alternating group. *J. Combinat. Theor.* Series B, **16** 86–96.

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